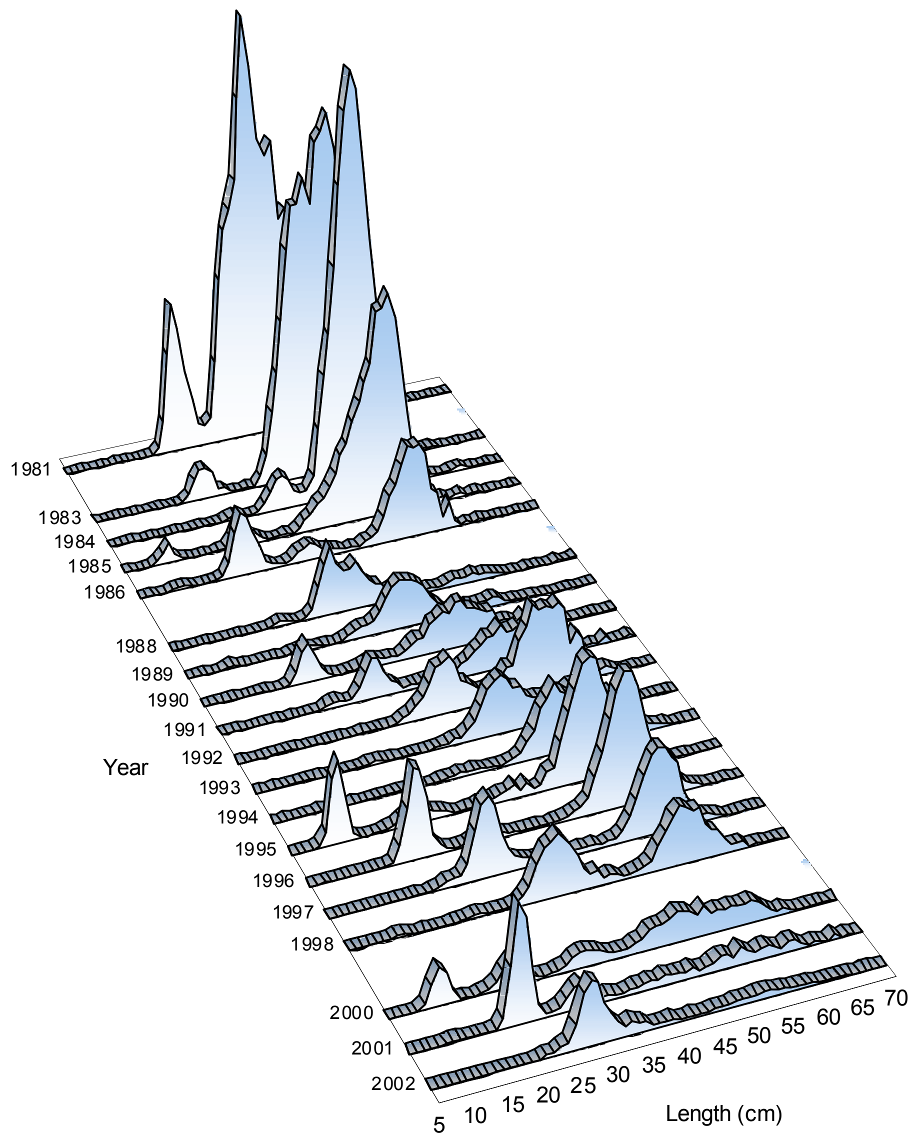


Figure 7. Biomass distribution by length of pollock in the Shelikof Strait EIT survey (1981-2002, except 1982,1987 and 1999).

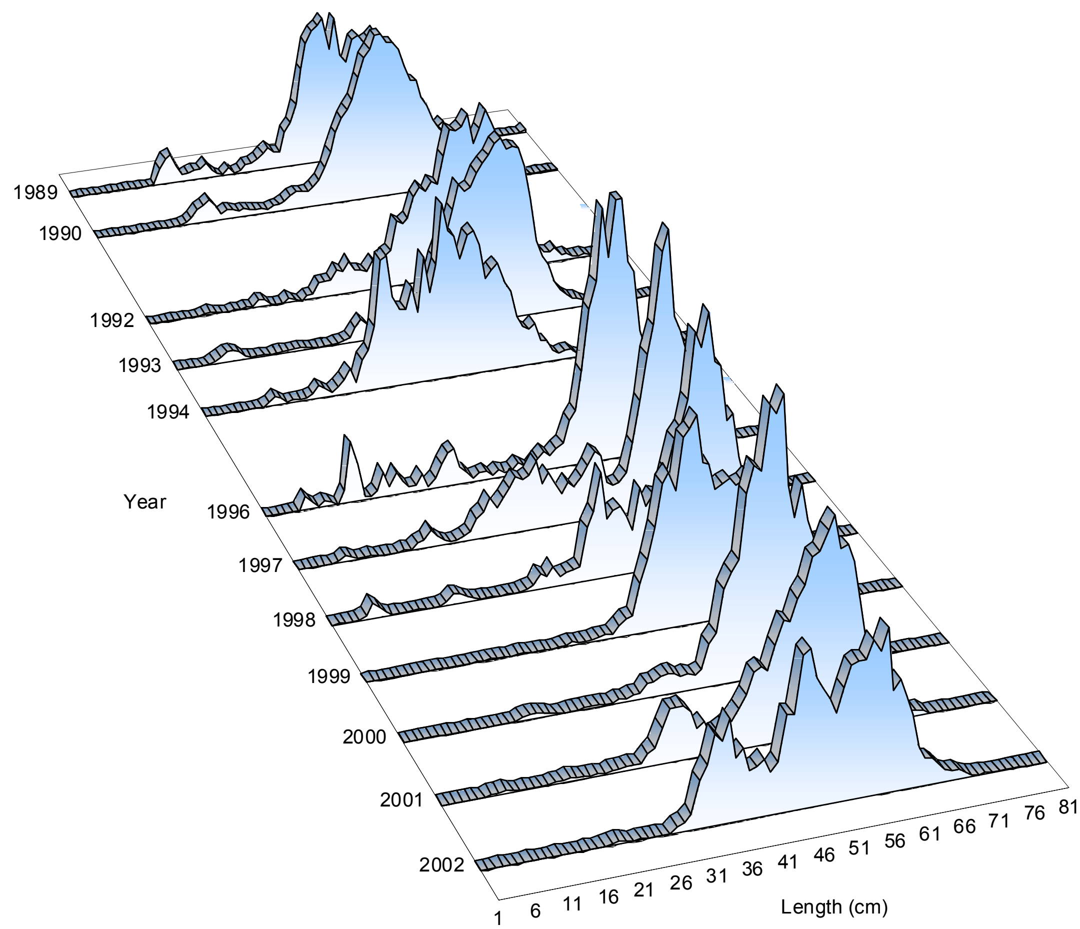


Figure 8. Length frequency of pollock in the ADF&G bottom trawl survey (1989-2002, except 1991 and 1995).

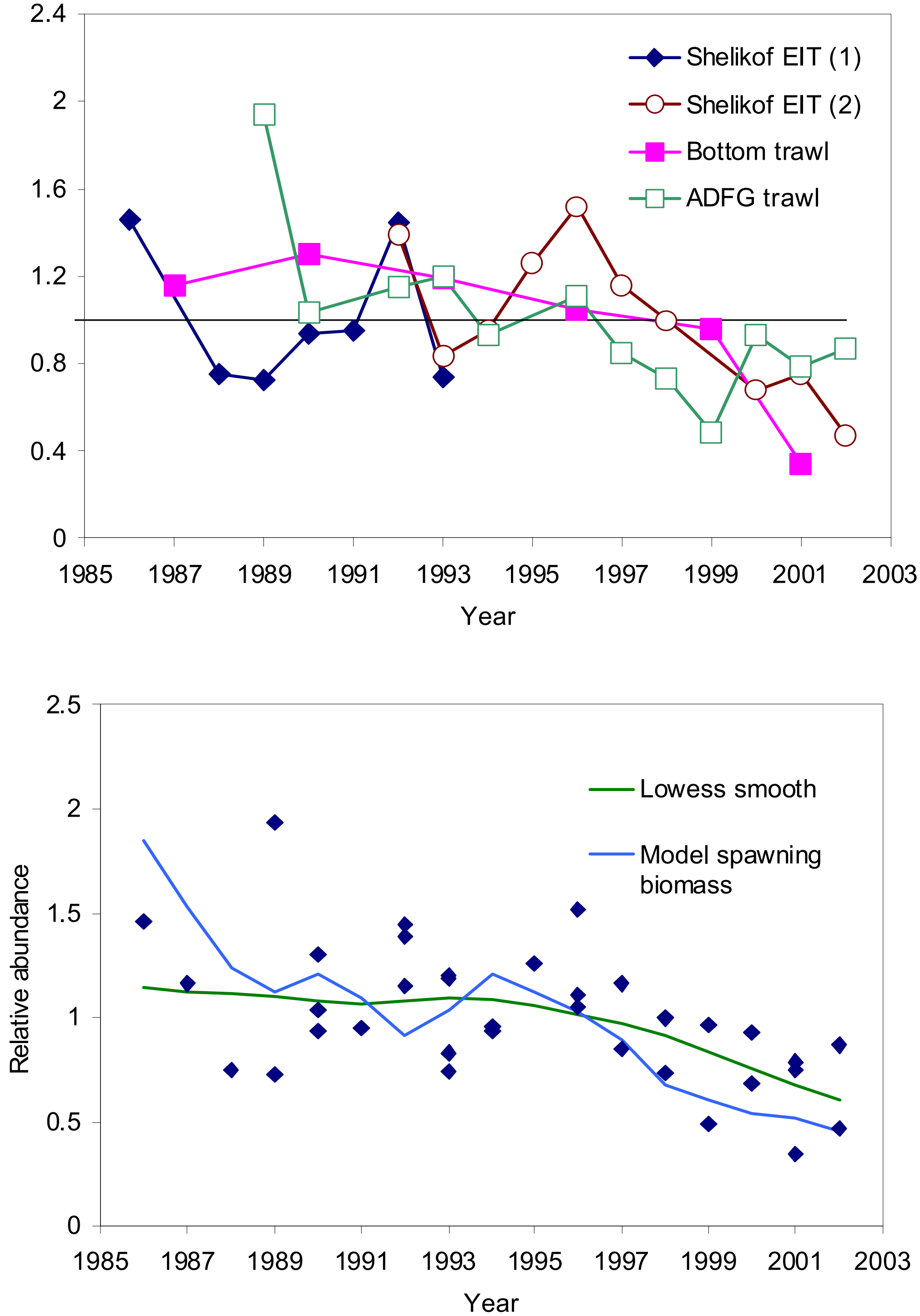


Figure 9. Trends in Gulf pollock biomass since 1986 for the Shelikof Strait EIT survey, the triennial bottom trawl survey, and the ADF&G coastal trawl survey. Each survey biomass estimate is standardized to the survey average since 1986. The Shelikof Strait EIT survey is split into separate time series corresponding to the two acoustic systems used for the survey. In the bottom panel, a lowess smooth (SPLUS 1993) of the same data is compared to the estimated biomass trend from the assessment model.

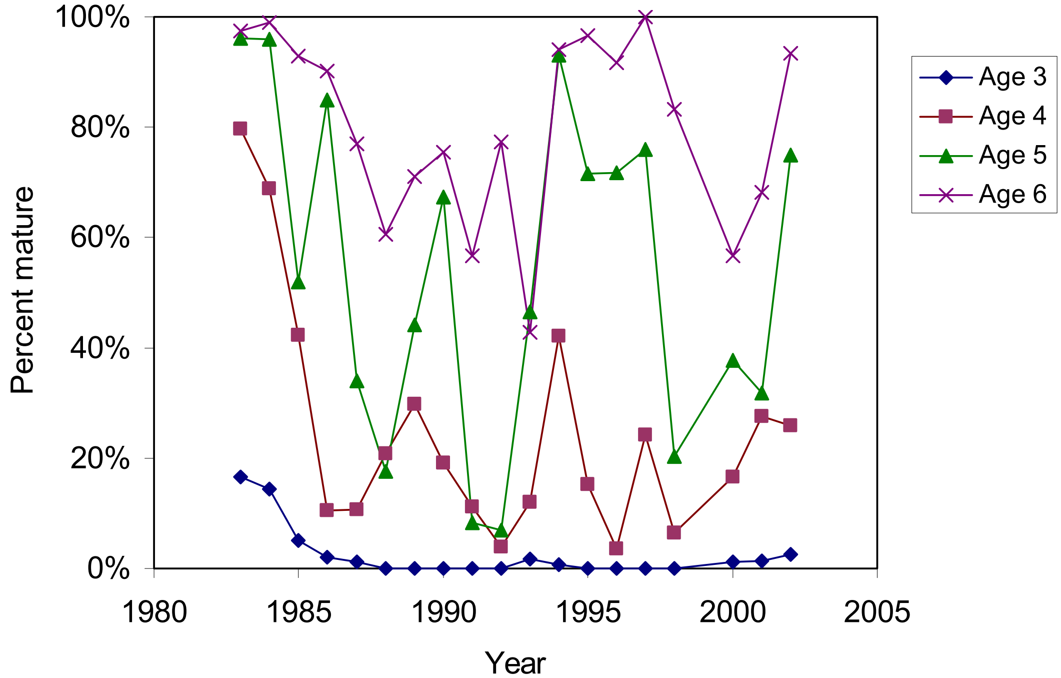


Figure 10. Percent mature female pollock ages 3-6 from winter EIT survey data in the Gulf of Alaska, 1983-2002.

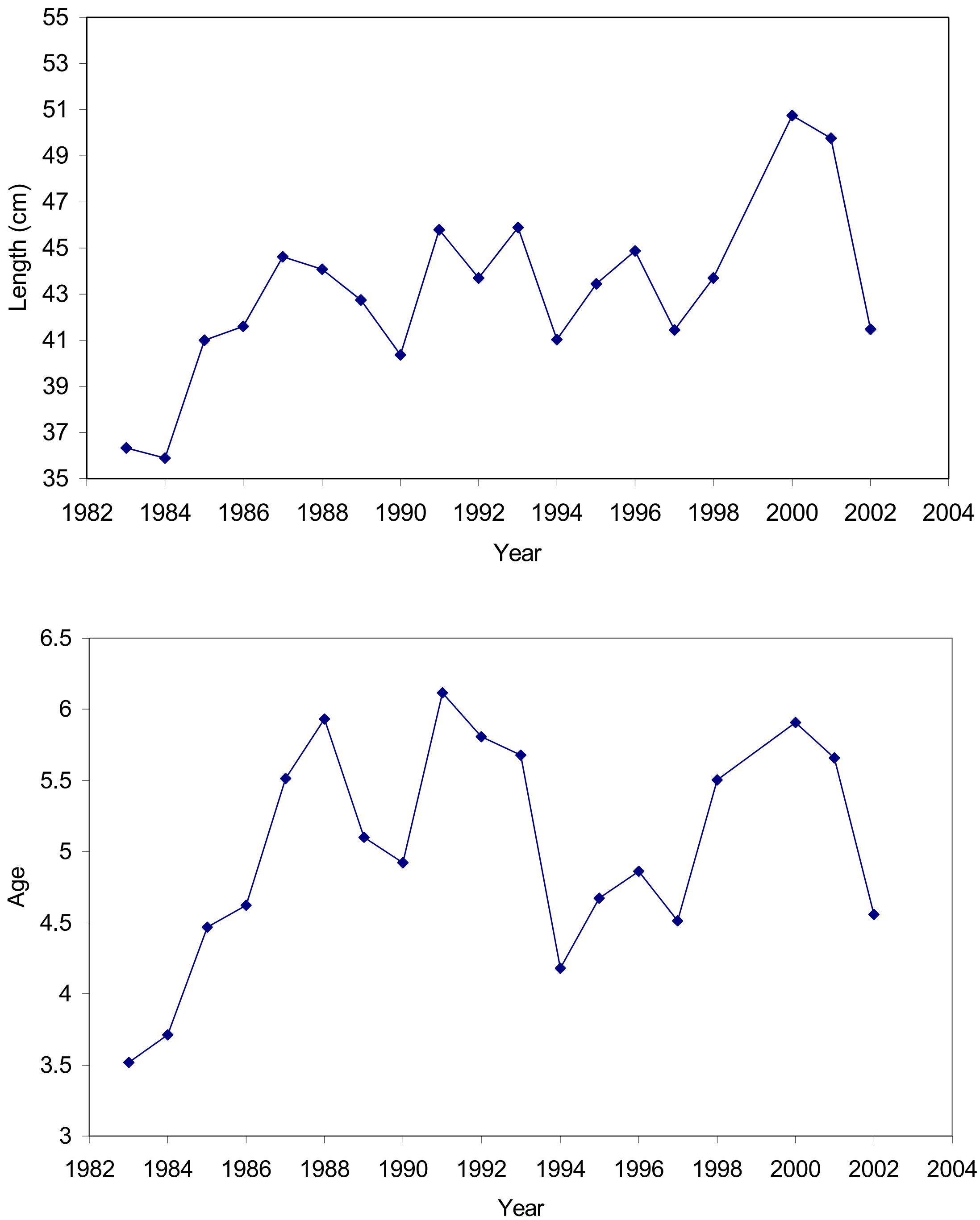


Figure 11. Age and length at 50% mature from annual logistic regressions for female pollock from winter EIT survey data in the Gulf of Alaska, 1983-2002.

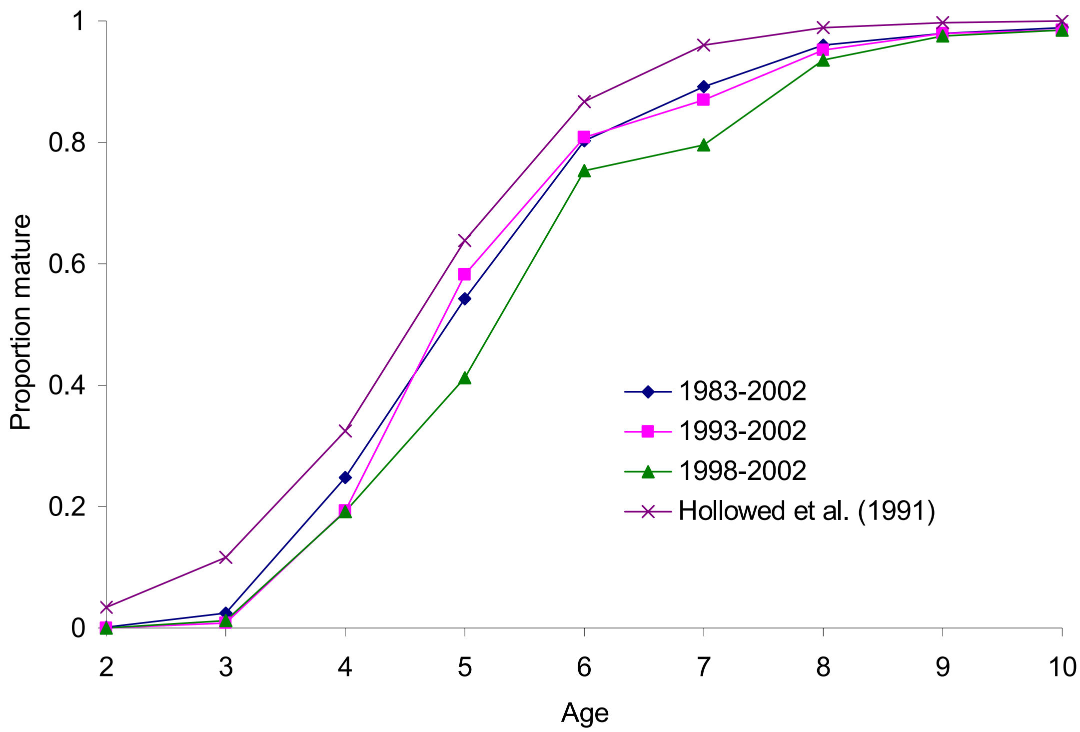


Figure 12. Maturity at age for Gulf of Alaska female pollock obtained by averaging the annual proportion mature at age for different ranges of years. The estimated maturity at age for Hollowed et al. (1991) is also shown.

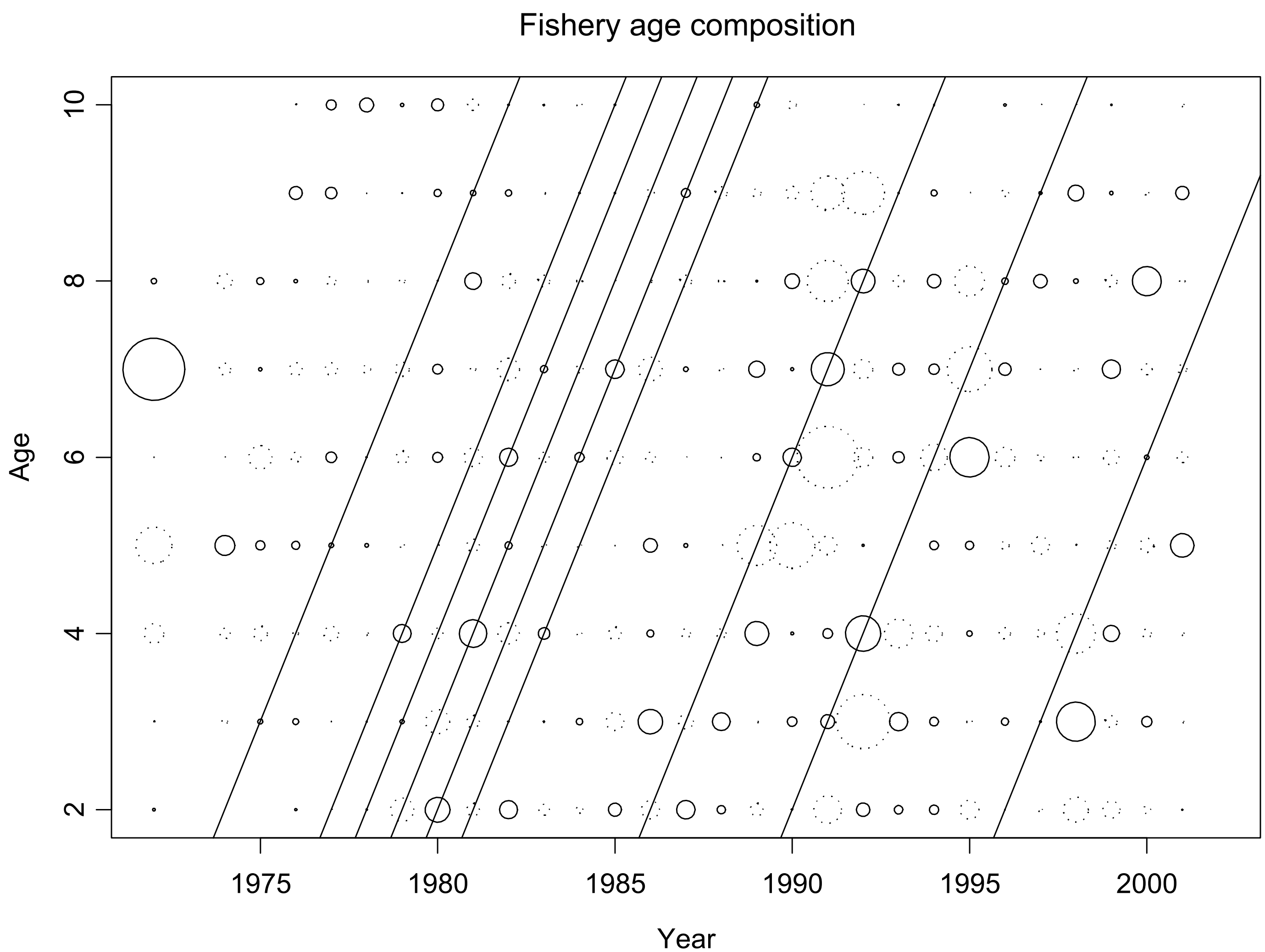
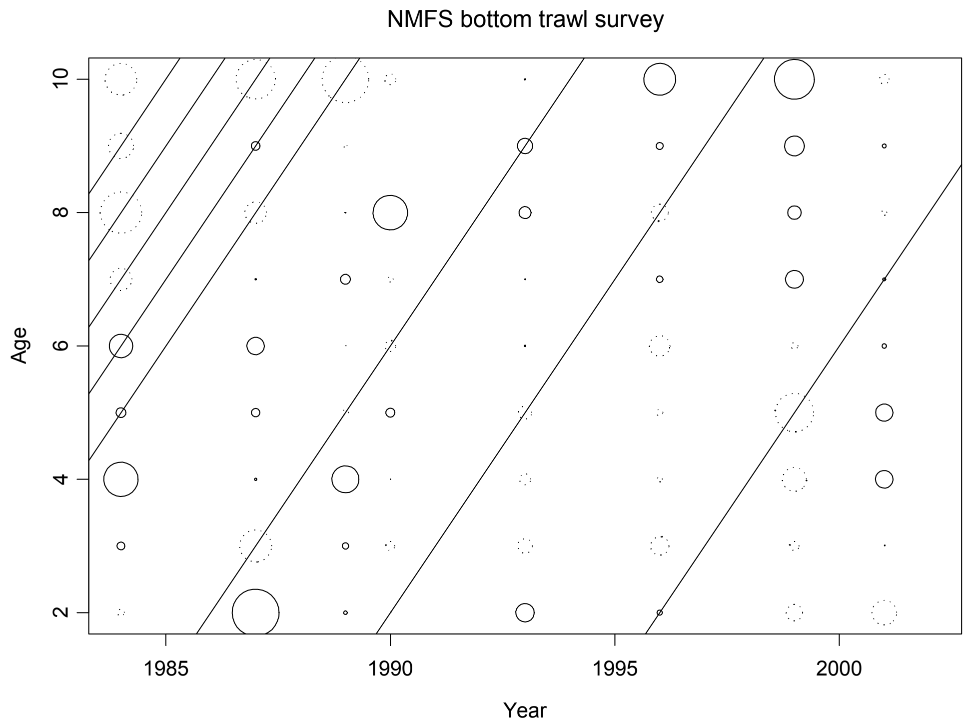


Figure 13. Residuals from Model 2 for fishery age composition (1972-2001). Circle diameters are proportional to the magnitude of the residual. Circles drawn with dotted lines indicate negative residuals. Diagonal lines show the strong year classes (1972, 1975, 1976, 1977, 1978, 1979, 1984, 1988, and 1994).



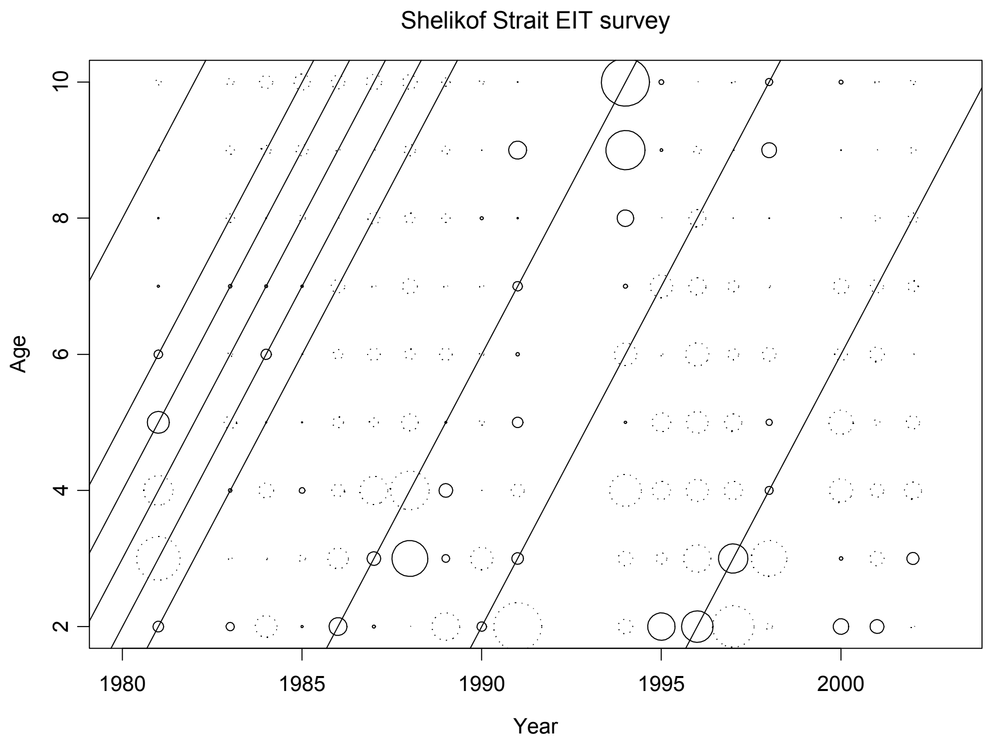


Figure 14. Residuals from Model 2 for the Shelikof Strait EIT survey age composition (top) and NMFS bottom trawl age composition (bottom). Circle diameters are proportional to the magnitude of the residual. Circles drawn with dotted lines indicate negative residuals. Diagonal lines show the strong year classes (1972, 1975, 1976, 1977, 1978, 1979, 1984, 1988, and 1994).

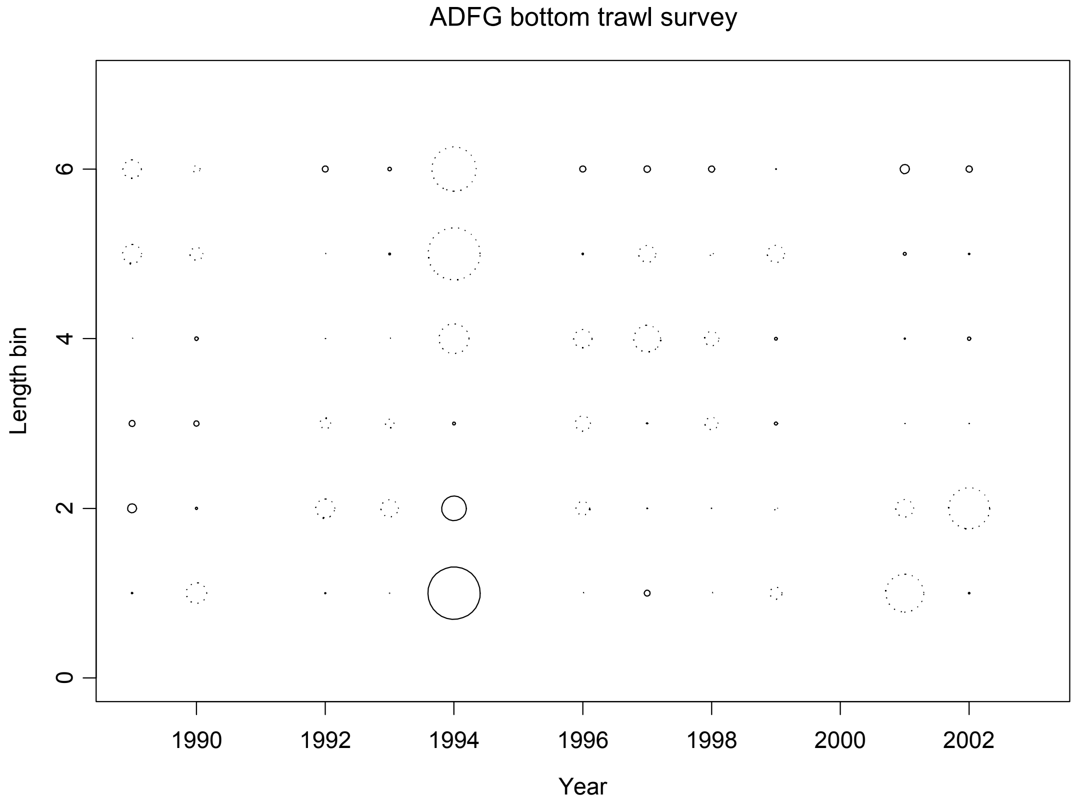
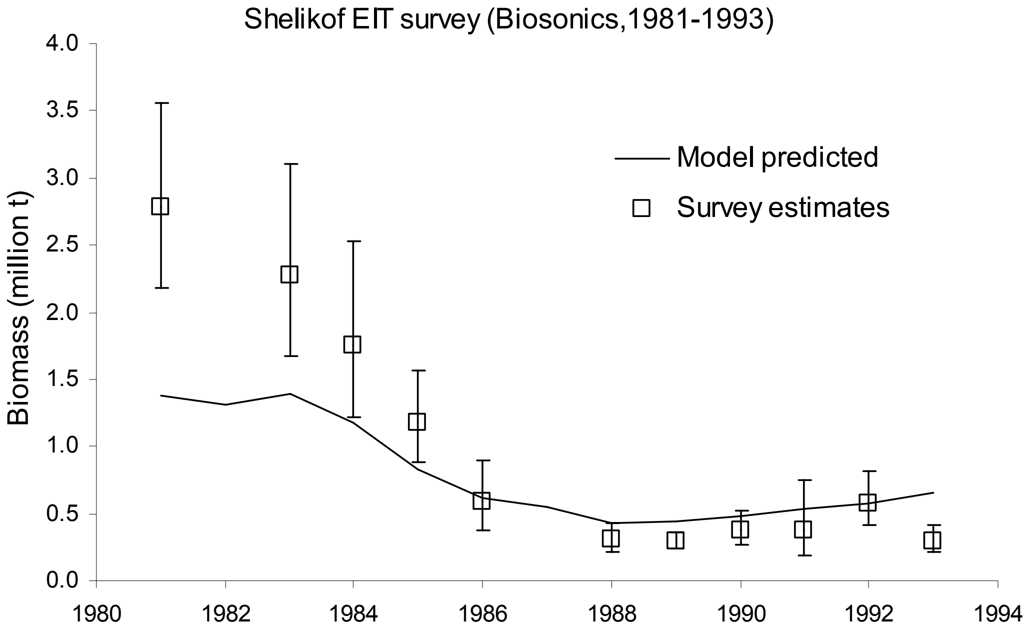


Figure 15. Residuals from Model 2 for the ADF&G survey length composition.



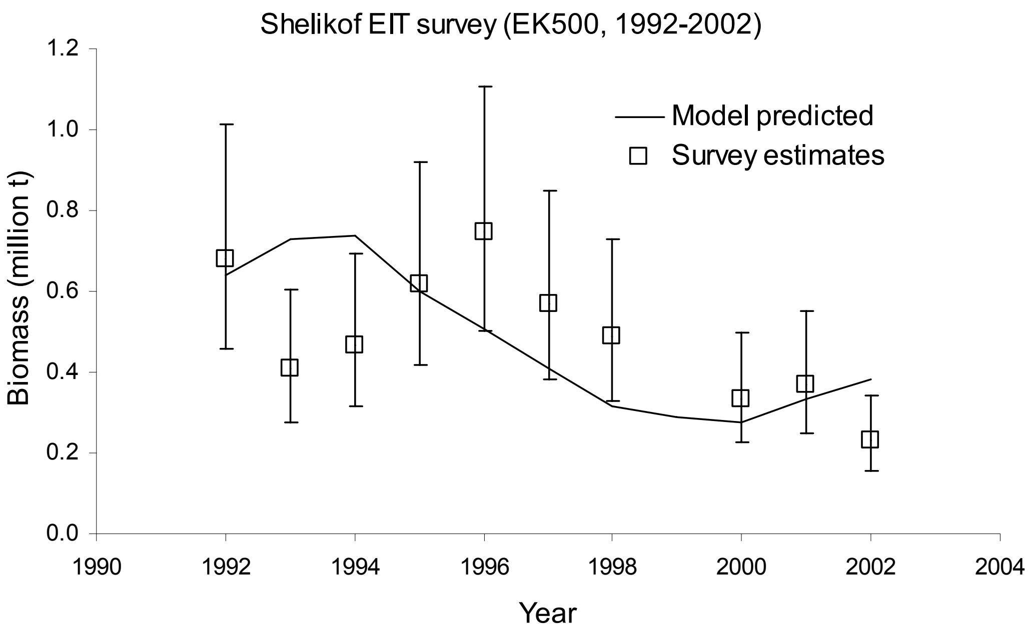


Figure 16. Model predicted and observed survey biomass for the Shelikof Strait EIT survey. The Shelikof EIT survey is modeled with two catchability periods corresponding to the two acoustic systems used for the survey. Error bars indicate plus and minus two standard deviations for the survey biomass estimate. Since variance estimates are unavailable for EK500 biomass estimates, an assumed CV of 0.2 is used in the assessment model.

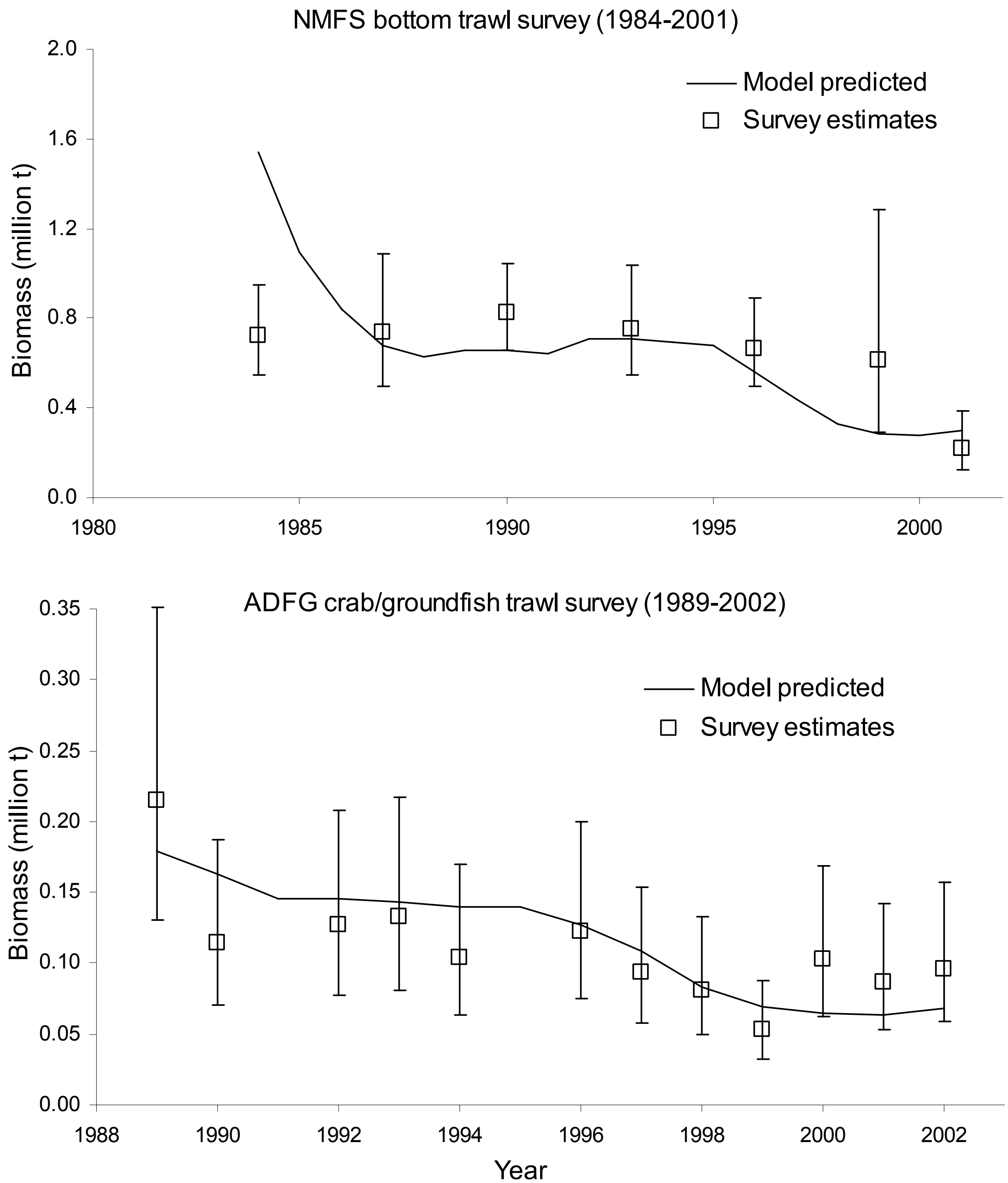


Figure 17. Model predicted and observed survey biomass for the NMFS bottom trawl survey (top panel), and the ADFG crab/groundfish survey (bottom panel). Error bars indicate plus and minus two standard deviations for the survey biomass estimate. Since variance estimates are unavailable for ADFG biomass estimates, an assumed CV of 0.25 is used in the assessment model.

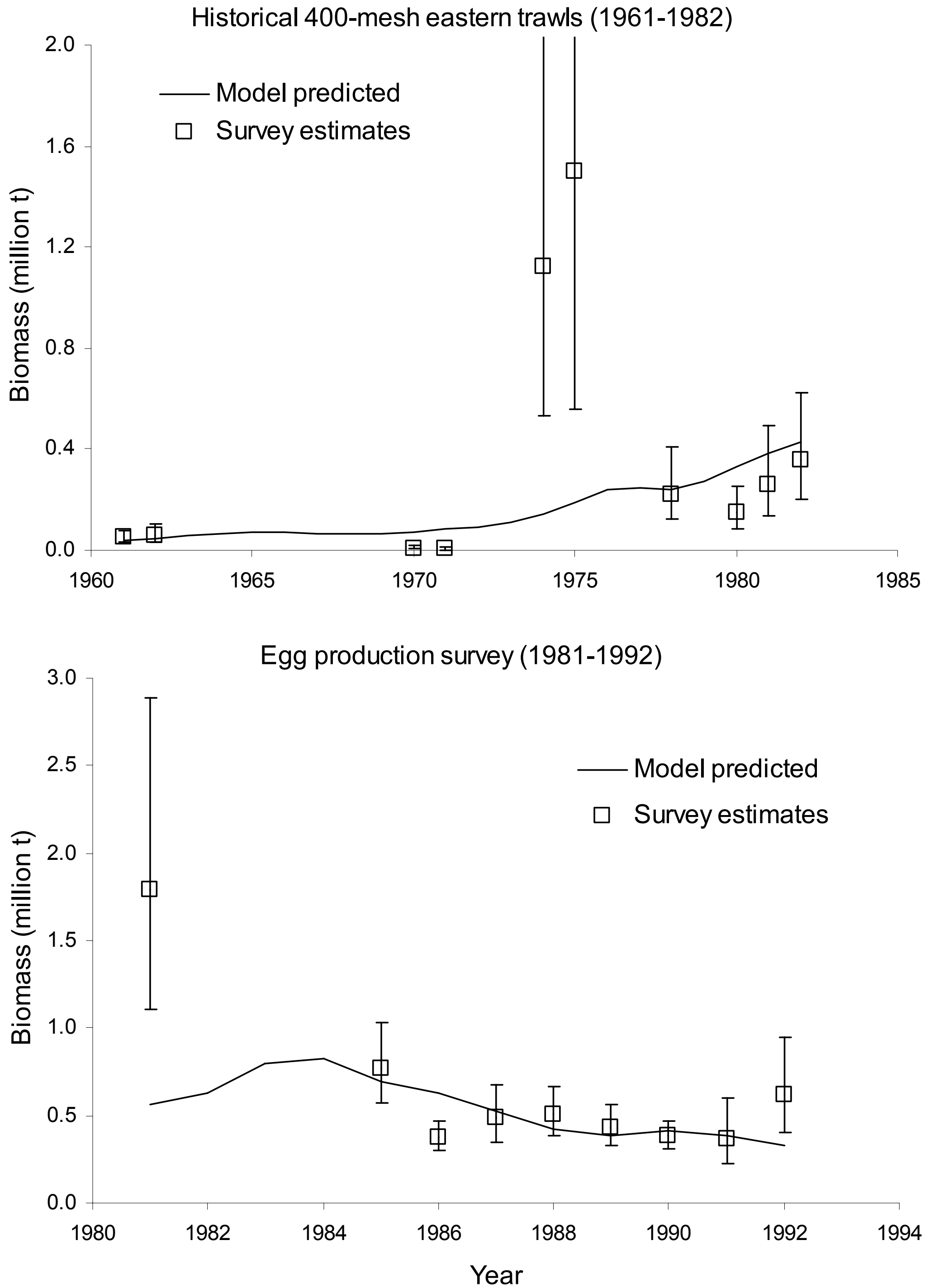


Figure 18. Model predicted and observed survey biomass for the historical 400-mesh eastern trawl surveys (top panel), and the egg production survey. Error bars indicate plus and minus two standard deviations for the survey biomass estimate.

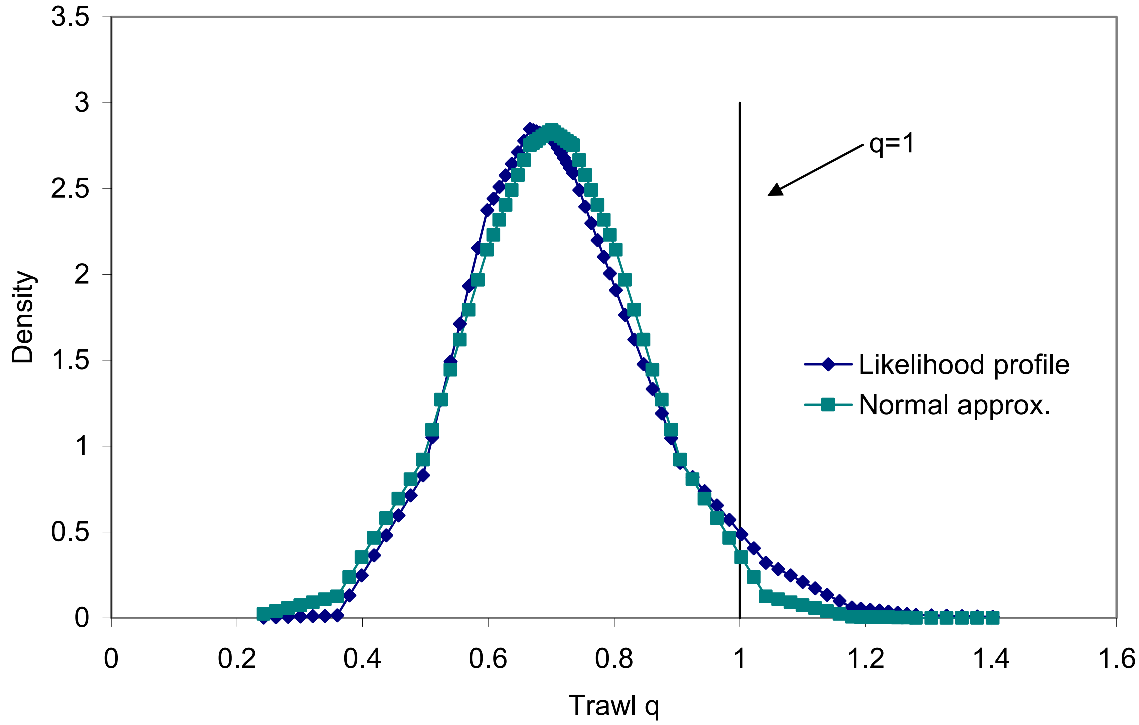


Figure 19. Uncertainty in the catchability coefficient for the NMFS trawl survey derived from likelihood profiling for Model 1.

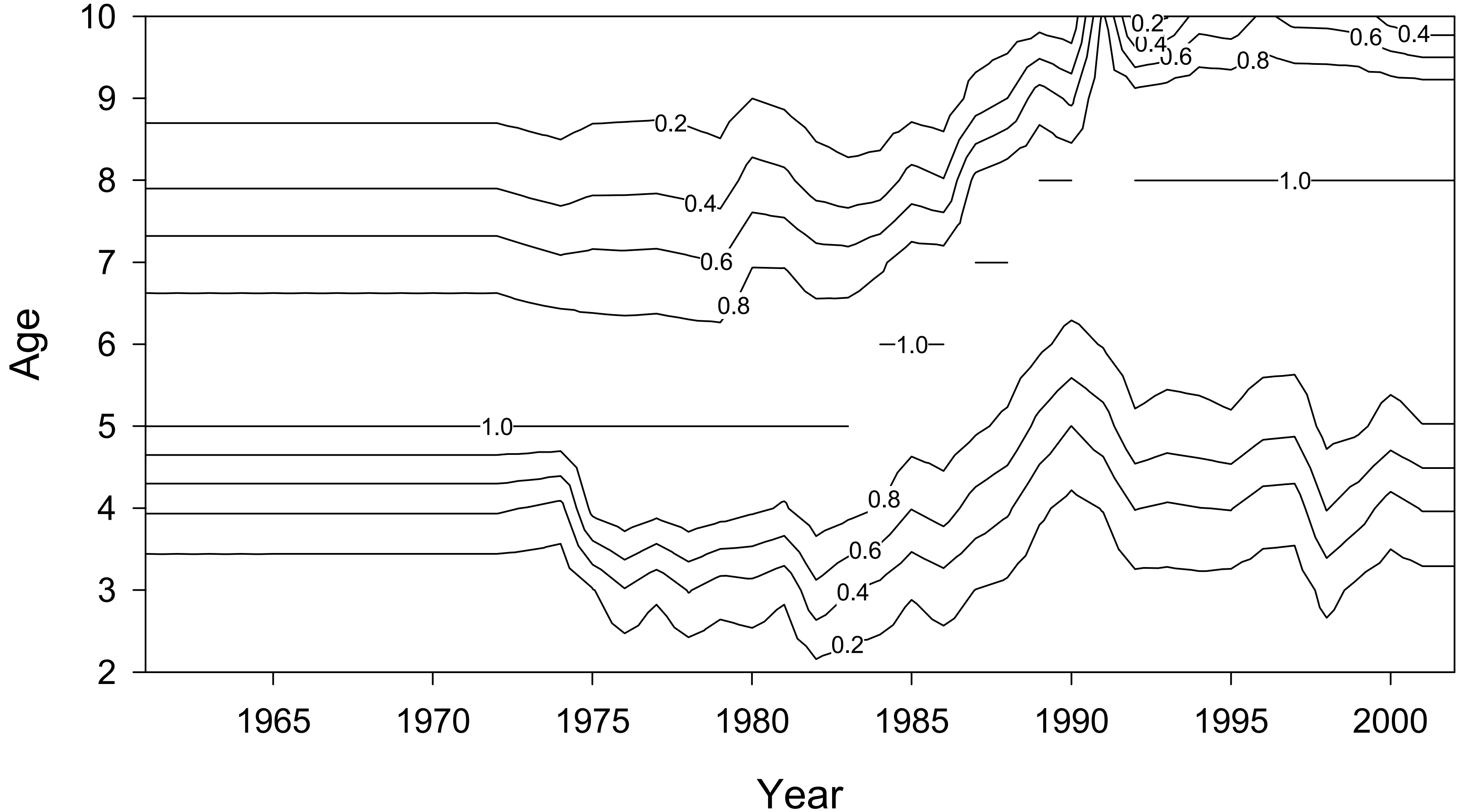


Figure 20. Estimates of time-varying fishery selectivity for Gulf of Alaska pollock.

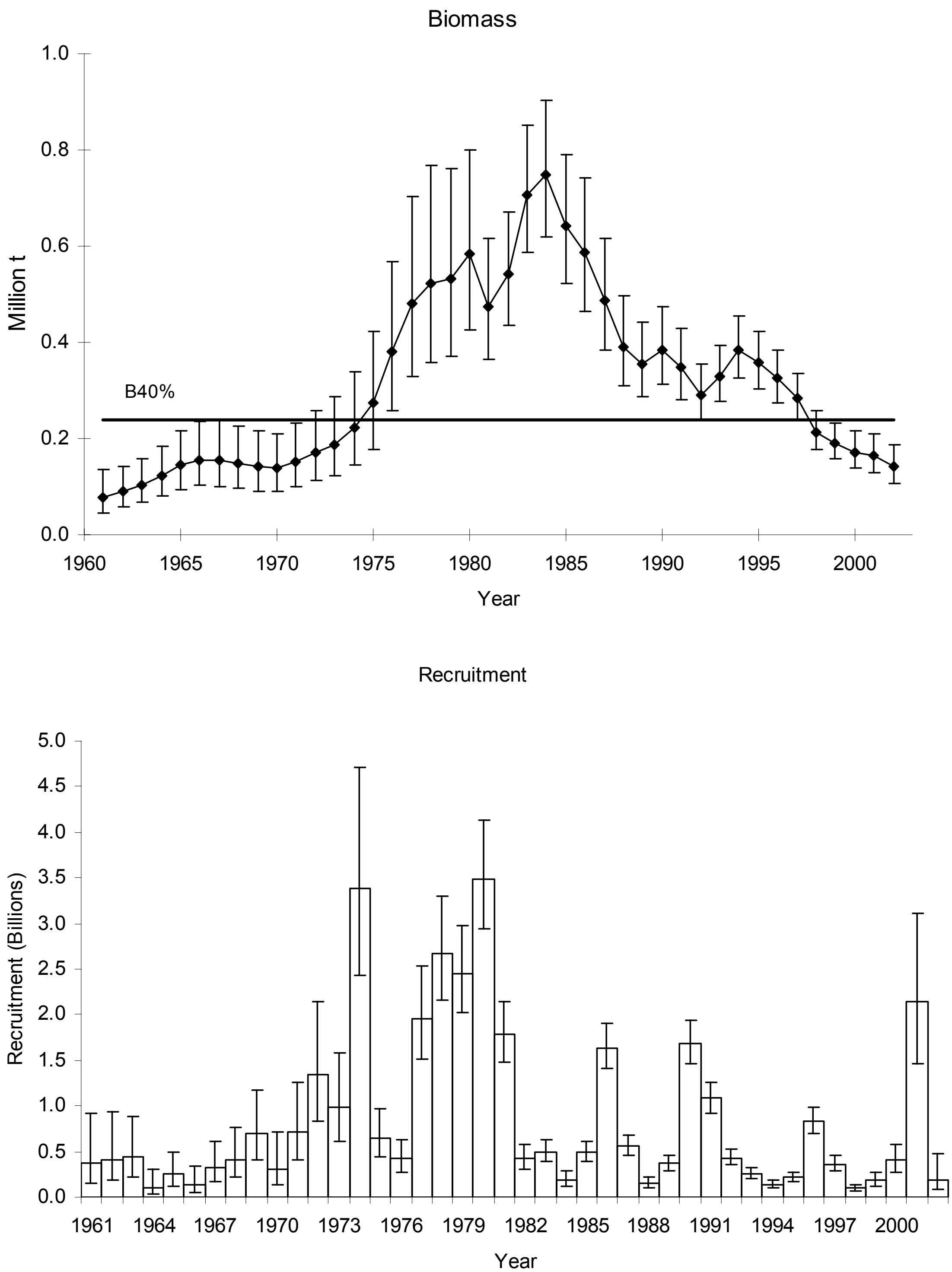


Figure 21. Estimated time series of Gulf of Alaska pollock spawning biomass (million t, top panel) and age-2 recruitment (billions of fish, bottom panel) from 1961 to 2002. Vertical bars represent two standard deviations. The B40% line represents the current estimate of this benchmark.

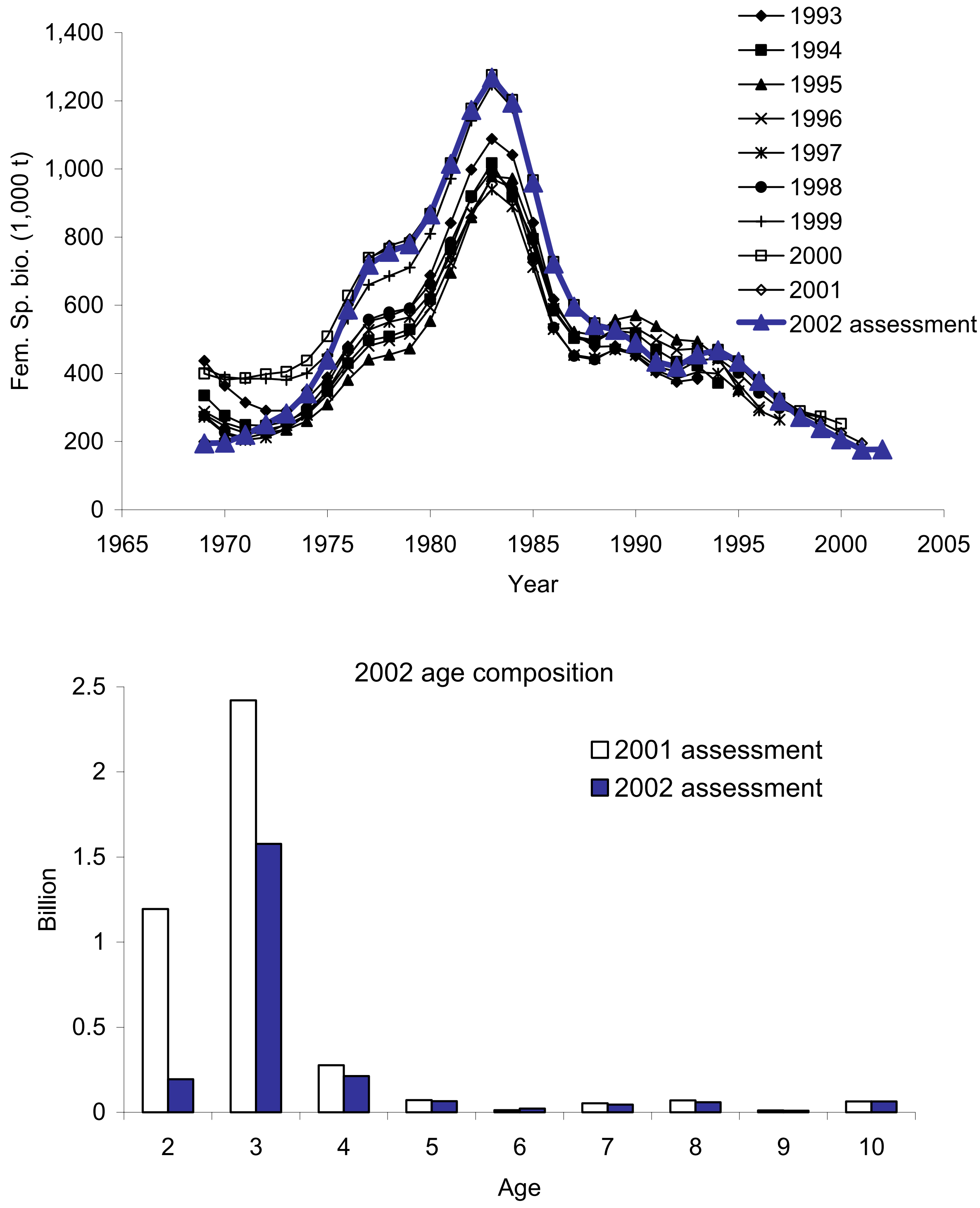


Figure 22. Retrospective plot of estimated Gulf of Alaska pollock female spawning biomass for stock assessments in the years 1993-2002 (top panel). For this figure, the time series of female spawning biomass for the 2002 assessment was calculated using the weight at age used in previous assessments to facilitate comparison. The bottom panel shows the estimated age composition in 2002 from the 2001 and 2002 assessments.

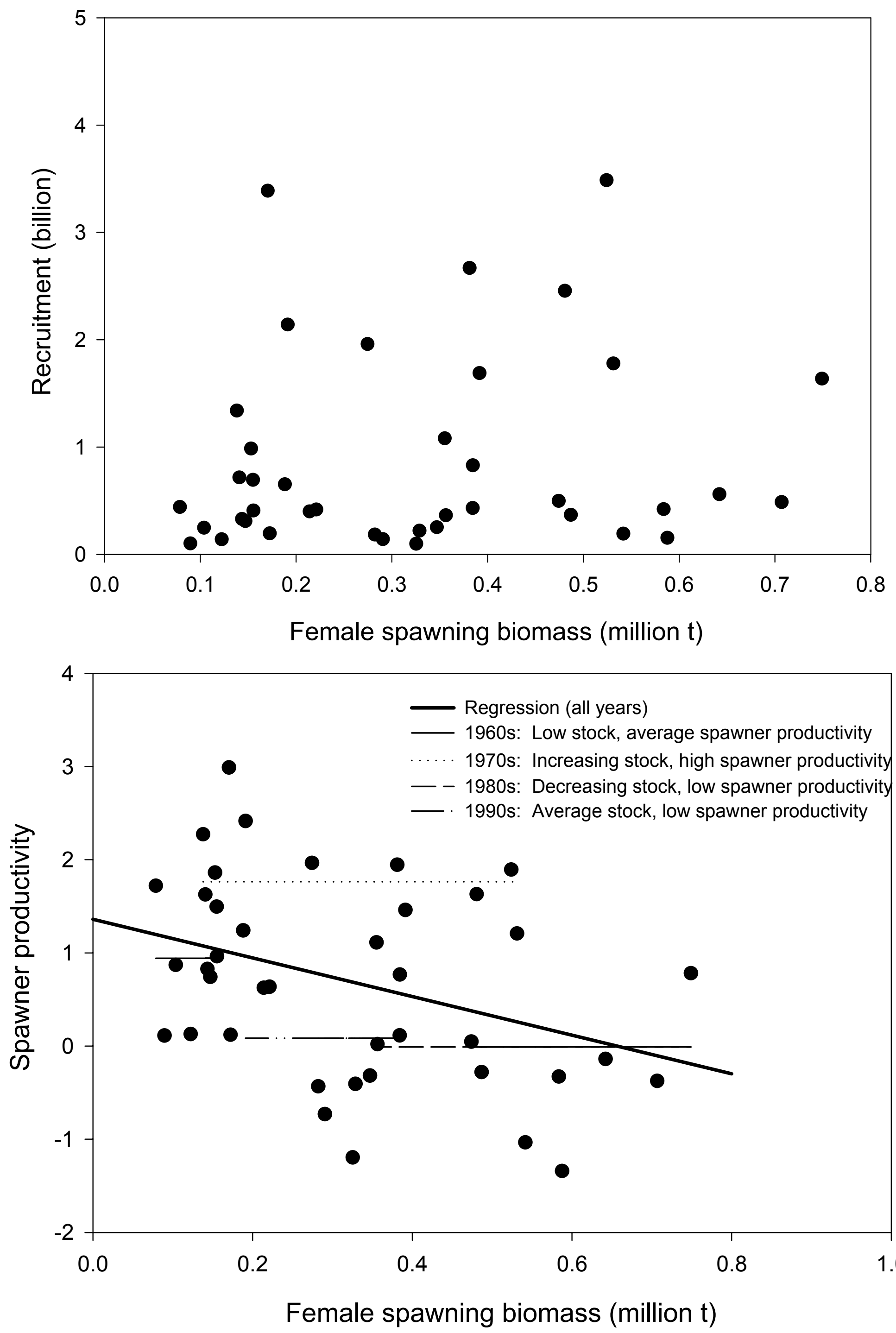


Figure 23. Gulf of Alaska pollock recruitment as a function of female spawning biomass (top). Spawner productivity $\log(R/S)$ in relation to female spawning biomass (bottom). The Ricker stock-recruit curve is linear in a plot of spawner productivity against spawning biomass. Horizontal lines indicate the mean spawner productivity for each decade within the range of spawning biomass indicated by the endpoints of the lines.

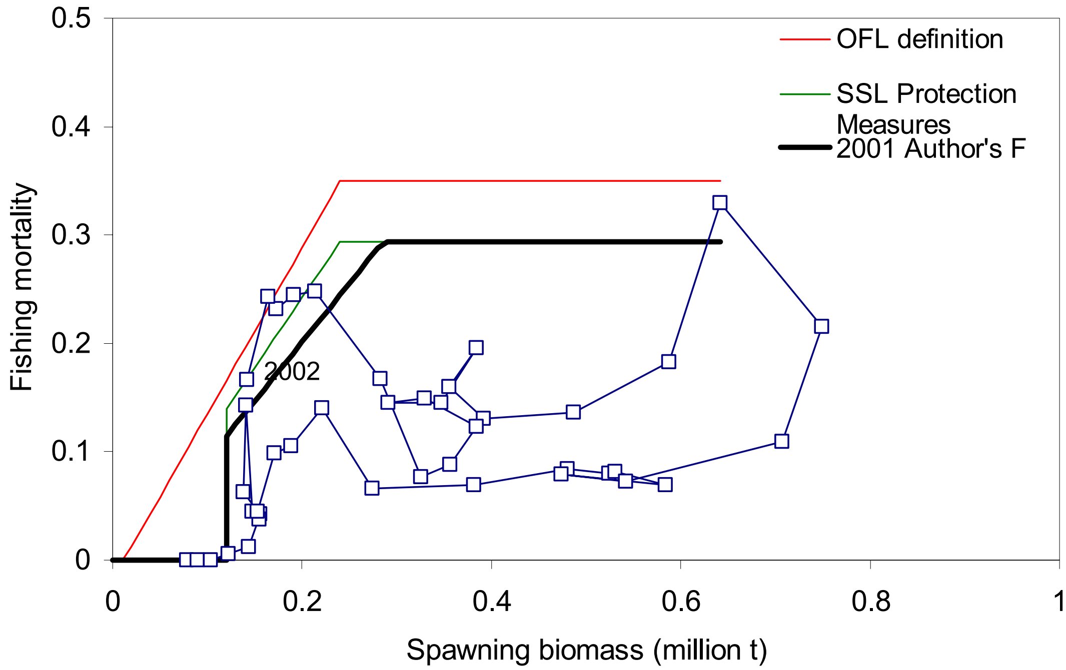


Figure 24. Estimates of Gulf of Alaska pollock spawning biomass and fishing mortality (1961-2002). The OFL definition and maximum permissible ABC are based on current estimates of fishery selectivity, maturity at age, weight at age, and mean recruitment. Because these estimates change as new data become available, this figure cannot be used to evaluate management performance relative to reference levels (F_{ABC}, F_{OFL} and $B_{40\%}$).

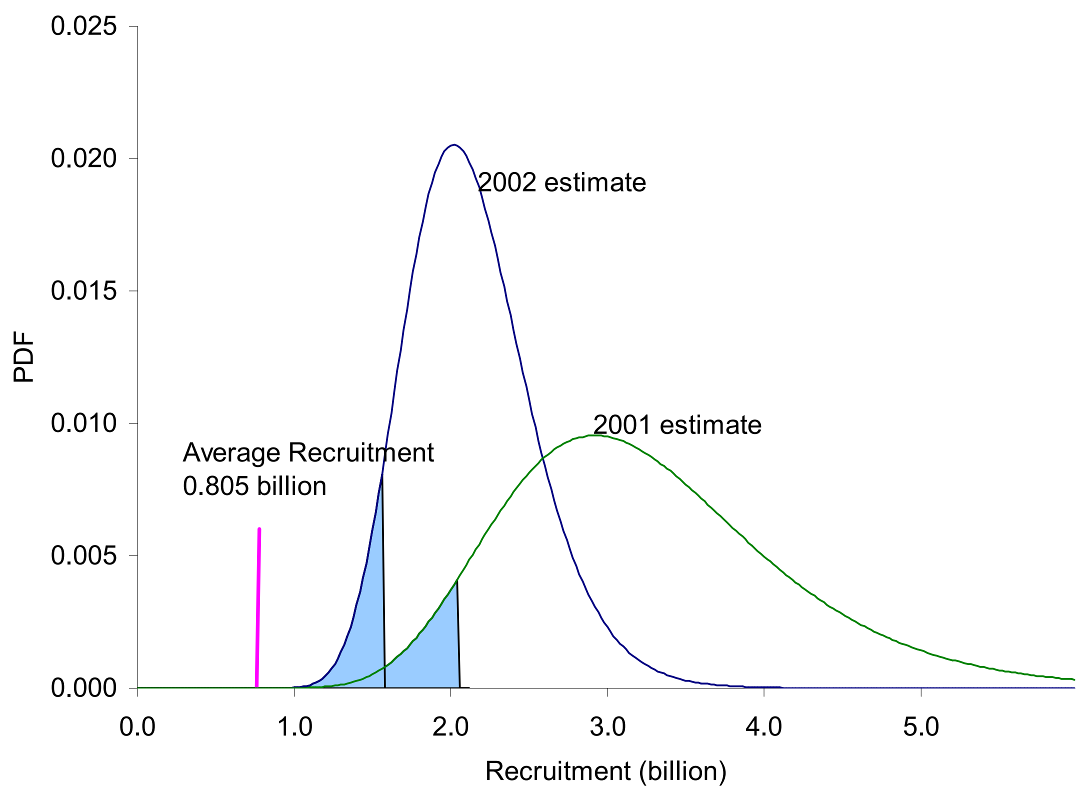


Figure 25. Uncertainty in the estimate of age-2 abundance of the 1999 year class in 2001 and 2002.

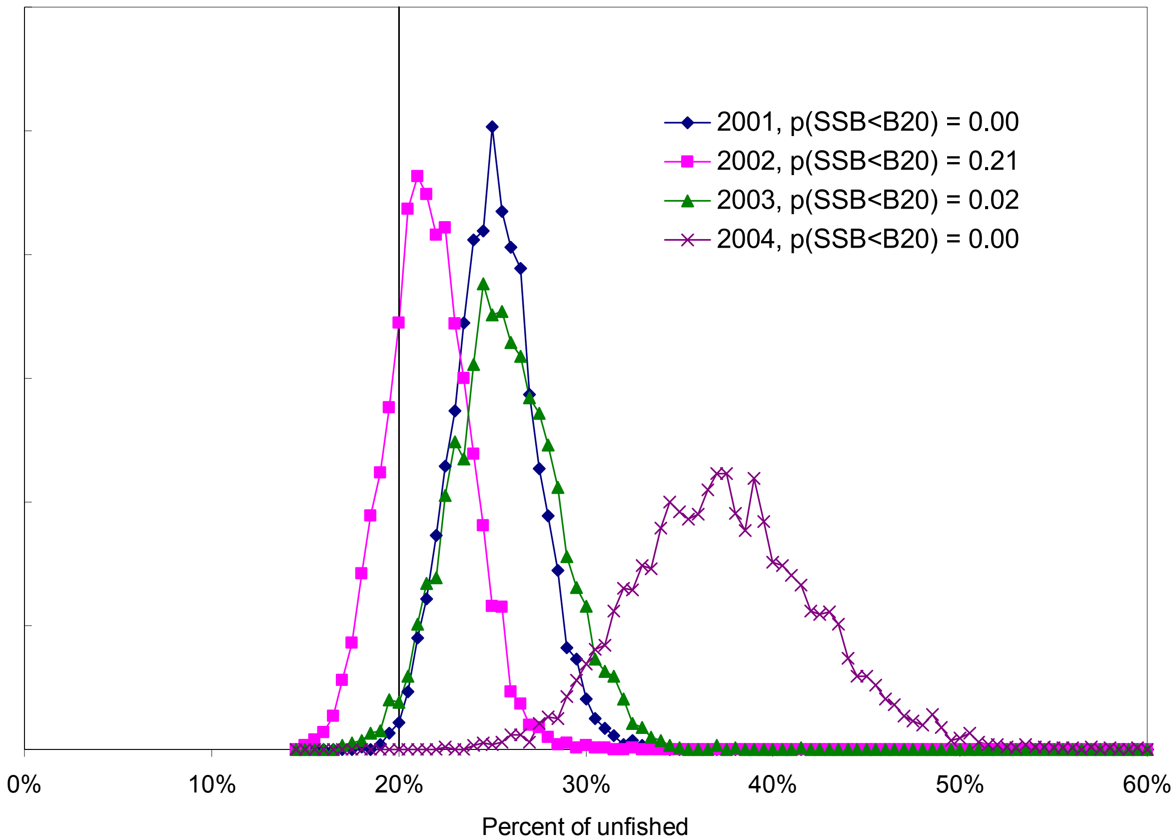


Figure 26. Uncertainty in spawning biomass in 2001-2004 based on a thinned MCMC chain from the joint marginal likelihood for Model 2 where catch in 2003 is fixed at the author's recommended ABC. In 2004, catch is derived from the MCMC estimate of spawning biomass in 2004 and the author's recommended fishing mortality schedule.

Appendix A: Southeast Alaska pollock

Bottom trawl surveys indicate a substantial reduction in pollock abundance east of 140° W. lon. Stock structure in this area is poorly understood. Bailey et al. (1999) suggest that pollock metapopulation structure in southeast Alaska is characterized by numerous fiord populations. In the 1996 and 1999 bottom trawl surveys, higher pollock CPUE in southeast Alaska occurred primarily from Cape Ommaney to Dixon Entrance, where the shelf is more extensive. Pollock size composition in the 1993, 1996 and 1999 surveys was dominated by smaller fish (<40 cm) (Martin 1997). These juveniles are unlikely to influence the population dynamics of pollock in the central and western Gulf of Alaska. Ocean currents are generally northward in this area, suggesting that juvenile settlement is a result of spawning further south. Spawning aggregations of pollock have been reported from the northern part of Dixon Entrance (Saunders et al. 1988).

Historically, there has been little directed fishing for pollock in southeast Alaska (Fritz 1993). During 1991-2000, pollock catch the Southeast and East Yakutat statistical areas averaged 20 t (Table 2). The current ban on trawling east of 140° W. lon. prevents the development of a trawl fishery for pollock in Southeast Alaska.

Pollock biomass estimates from the bottom trawl survey are highly variable, in part due to year-to-year differences in survey coverage. The 1996 and 1999 surveys had the most complete coverage of shallow strata in southeast Alaska, and indicate that stock size is approximately 25-75,000 t (Fig. 27). We recommend placing southeast Alaska pollock in Tier 5 of NPFMC harvest policy, and basing the ABC and OFL on natural mortality (0.3) and the biomass >30 cm (a proxy for exploitable biomass) for the 1999 survey. Because the NMFS trawl survey in 2001 did not extend to southeast Alaska, no new survey information will be available until 2003. Biomass in southeast Alaska was estimated by splitting survey strata and CPUE data in the Yakutat INPFC area at 140° W. lon. and combining the strata east of the line with comparable strata in the Southeastern INPFC area. This gives a **2002 ABC of 6,460 t** (28,709 t * 0.75 M), and a **2002 OFL of 8,613 t** (28,709 t * M).

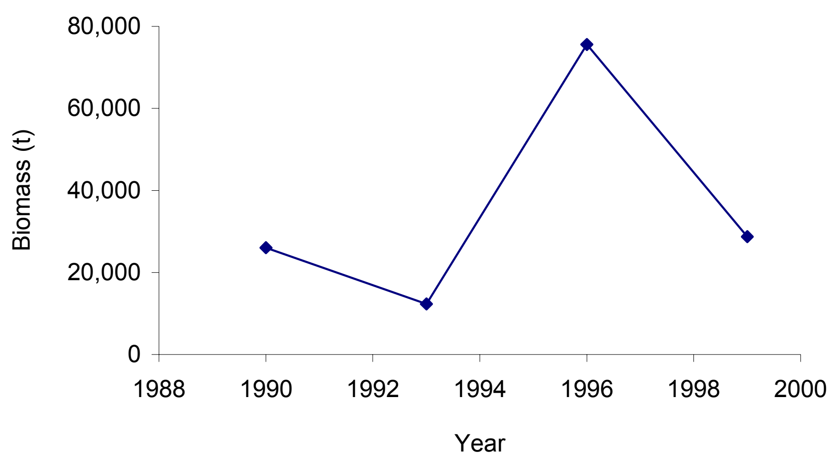


Figure 27. Pollock biomass trend in southeast Alaska from NMFS bottom trawl surveys in 1990-1999.

Appendix B: Seasonal distribution and apportionment of walleye pollock among management areas in the Gulf of Alaska

Since 1992, the Gulf of Alaska pollock TAC has been apportioned between management areas based on the distribution of biomass in groundfish surveys. Both single species and ecosystem considerations provide the rationale for TAC apportioning. From an ecosystem perspective, apportioning the TAC will spatially distribute the effects of fishing on other pollock consumers (i.e., Steller sea lions), thus reducing the overall intensity of any averse effects. From the perspective of the pollock population, apportioning the TAC ensures that no smaller component of the stock experiences higher mortality than any other. Although no sub-stock units of pollock have yet been identified in the Gulf of Alaska, it would be precautionary to manage the fishery so that if these sub-units do exist they would not be subject to high fishing mortality. Protection of sub-stock units would be most important during spawning season, when they are spatially separated. The Steller Sea Lion Protection Measures implemented in 2001 require apportionment of pollock TAC based on the seasonal distribution of biomass. Although spatial apportionment is intended to reduce the potential impact of fishing on endangered Steller Sea Lions, it is important to recognize that apportioning the TAC based on an inaccurate or inappropriate estimate of biomass distribution could have adverse impacts, both on pollock population itself, and on species that depend on pollock.

Walleye pollock in the Gulf of Alaska undergo an annual migration between summer foraging habitats and winter spawning grounds. Since surveying effort has been concentrated during the summer months and prior to spawning in late winter, the dynamics and timing of this migration are not well understood. Regional biomass estimates are highly variable, indicating either large sampling variability, large interannual changes in distribution, or, more likely, both. There is a comprehensive survey of the Gulf of Alaska in summer, but surveying during winter has focused on the Shelikof Strait spawning grounds, with limited surveying of the Shumagin Islands spawning grounds in the middle to late 1990s, and additional exploratory surveying along the shelf break in the early 1990s. It is important to make the best use of limited information on biomass distribution from surveys when apportioning the TAC. Here we summarize available information on 1) the timing of pollock migration to and from spawning areas, 2) the winter distribution of pollock in the Gulf of Alaska, and 3) the summer distribution of pollock.

Timing of pollock movement to and from spawning areas

There is limited information concerning the timing of pollock movement to and from spawning areas in the Gulf of Alaska. Replicated surveys of pollock in the 1980s show a gradual buildup of biomass in Shelikof Strait, followed by a rapid decline after the peak of spawning between 15 March and 1 April (Fig. 28). Pollock distribution in the Gulf of Alaska trawl survey, which begins in June, shows that pollock have migrated to their summer feeding habitat, with relatively low biomass in areas where spawning occurred earlier in the year, such as upper Shelikof Strait. Since approximately 65% of GOA pollock spawn in Shelikof Strait, there would have to be significant migration of adult fish both to the east side of Kodiak Island and to the southwest after spawning to produce the pollock distribution in summer surveys.

Since the earliest survey in Shelikof Strait began on February 21, no survey information exists on pollock distribution between January 20, when the current A season fishery opens, and late February. Beginning in 2000, there have been two fishery openings (A and B seasons) prior to peak spawning. The distribution of fishing around Kodiak Island in these two openings has been very different, suggesting that the pollock have not attained their typical spawning distribution by January 20 (Figs. 29-31). During the A-season in 2000 and 2001, catches were on Kodiak Island side of the strait near the northern end. In the B-season, fishing was concentrated further down the strait along the Alaska Peninsula side. This suggests that, at least in some years, pre-spawning pollock may migrate around the north end of Kodiak Island to Shelikof Strait. In 2002, a very different pattern was observed, with no pollock catches in the northern part of Shelikof Strait, and large catches along the shelf break outside of Shelikof Strait during in the B season. In the Shumagin area, fishing patterns were similar in the A and the B seasons, which is consistent with survey results indicating earlier spawning in this area.

Previous recommendations for apportioning the A-season TAC between management areas were based on the assumption that the pollock stock on January 20 has the same spatial distribution as the mean distribution on the spawning grounds in mid-March. The experience of the fishing fleet since 2000 suggests that this assumption should be reconsidered, at least for areas 620 and 630. We present three options for apportioning the TAC between 620 and 630 for the A season fishery (Fig. 32). The first option assumes that the migration has not started by the A season opening, and apportions the TAC between areas 620 and 630 by assuming the summer percentage in 630 and obtaining 620 percent by subtraction. This results in a percent apportionment of 24.73%, 45.23% and 30.05% in areas 610, 620, and 630 respectively. The second option is based on the estimated winter distribution, giving 24.73%, 66.46% and 8.81%, while the last option takes the midpoint of these two assumptions, giving 24.73%, 55.84% and 19.43% for areas 610, 620, and 630 respectively. Note that the proportion in 610 remains the same in each option, reflecting the assumption that pollock targeted by the 610 fishery in A season are fish that will spawn in area 610.

Winter distribution

In winter, an annual acoustic survey in Shelikof Strait has been conducted since 1981. A significant portion of the remaining shelf and upper slope waters in the Gulf of Alaska west of Cape Suckling has been surveyed at least once during winter by exploratory surveys and surveys with shorter time series. No acoustic survey has been comprehensive, covering all areas where pollock could potentially occur. Therefore a “composite” approach was developed to use data from several different surveys. We used data from 1) Shelikof Strait surveys in 1992-2002, 2) surveys of the Shumagin Island area in 1995, 2001, and 2002 (Wilson et al. 1995, Guttormsen et al. 2001, 2002), and 3) an exploratory survey along the shelf break in 1990 (Karp 1990). Each of these surveys covered a non-overlapping portion of the Gulf of Alaska shelf and upper slope west of Cape Suckling. Surveys of the Shumagin Island area in 1994 and 1996 were not used in this analysis because most fish were in post-spawning condition, and replicated surveys of spawning pollock in Shelikof Strait indicate a rapid decline in abundance after peak spawning (Wilson 1994, Wilson et al. 1996).

The “composite” approach was to estimate the percent of the total stock surveyed during a particular survey by dividing the survey biomass by the estimated total biomass of pollock at spawning from the assessment model. The percent for each non-overlapping survey was added together to form a composite biomass distribution, which, with some luck, ought to be close to 100%. Model estimates of biomass at spawning took into account the total mortality between the start of the year and spawning, and used mean weight at age from Shelikof Strait surveys in 1992-2002.

Results indicate that an average of 63% of the pollock biomass was in Shelikof Strait in winter (Appendix table 1). For the Shumagin surveys in 1995, 2001, and 2002, 25% of the total stock biomass was surveyed on average. The sum of the percent biomass for all surveys was 97%, which may reflect sampling variability, interannual variation in spawning location, or differences in echo sounder/integration systems, but also suggests reasonable consistency between the aggregate biomass of pollock surveyed acoustically in winter and the assessment model estimates of abundance. After rescaling, the resulting average biomass distribution was 24.73%, 66.46%, and 8.81% in areas 610, 620, and 630.

Summer distribution

The NMFS bottom trawls is summer survey (typically extending from mid-May to mid-August). Because of large shifts in the distribution of pollock between management areas one survey to the next, and the high variance of biomass estimates by management area, Dorn et al. (1999) recommended that the apportionment of pollock TAC be based upon the four most recent NMFS summer surveys. The four-survey average was updated with 2001 survey results resulting in an average biomass distribution of 45.95%, 22.44%, 29.37%, and 2.25% in areas 610, 620, 630, and 640 (Fig. 33).

TAC Rollovers

In 2002, the low abundance of pre-spawning pollock in Shelikof Strait resulted in significant unharvested TAC in area 620 at the end of the A and B seasons. An underage or overage of TAC for a seasonal opening can be added to or subtracted from subsequent allowances provided the seasonal allowances do not exceed 30% of the total TAC. Historically, adjustments in the seasonal TACs have been made only within management areas. For example, this year, the underage in area 620 was rolled over into the TACs for C and D seasons in area 620, where summer bottom trawl surveys indicate that the abundance of adult pollock is much lower than in winter. To achieve the goal of the Steller Sea Lion Protection Measures to equalize the exploitation rates spatially and temporally, a better approach would be to accumulate the gulf-wide overage or underage at the end of the winter fisheries (i.e., the end of the B season), then apportion the tonnage between areas 610, 620, and 630 according to the summer biomass distribution.

Specific recommendations for plan team consideration

1. Consider alternative apportionment schemes for the A season fishery in areas 620 and 630

Option A. Summer survey biomass distribution in areas 620 and 630: 610, 24.73%; 620, 45.23%; and 630, 30.05%.

Option B. Winter survey biomass distribution in areas 620 and 630: 610, 24.73%; 620, 66.46%; and 630, 8.81%.

Option C. Midpoint between summer and winter distributions in areas 620 and 630: 610, 24.73%; 620, 55.84%; and 630, 19.43%.

2. Recommend that overage or underages at the end of the winter fisheries be accumulated for all areas and be redistributed to management areas according to the summer distribution of biomass.

Example calculation of 2003 Seasonal and Area TAC Allowances

Warning: This example is based on hypothetical TAC of 100,000 t.

1) Since no information is available on the seasonal distribution of pollock in area 640, use summer biomass distribution:

640 0.0225 x Total TAC = 2,250 t

2) Calculate seasonal apportionments of TAC for the A, B, C, and D seasons at 25 %, 25%, 25%, and 25 % of the remaining annual TAC west of 140° W lon.

A season 0.25 x (Total TAC – 2,250) = 24,437
B season 0.25 x (Total TAC – 2,250) = 24,437 t

C season	0.25 x (Total TAC – 2,250) = 24,437 t
D season	0.25 x (Total TAC – 2,250) = 24,437 t

3) For the A season, the allocation of TAC to areas 610, 620 and 630 is based on either the "composite" estimate of winter biomass distribution, or a blending of winter and summer distributions to reflect that pollock may not have completed their migration to spawning areas by Jan. 20, when the A season opens. In the example below, we use Option C which assumes that the migration is half completed.

610	0.2473 * 24,437 t = 6,043 t
620	0.5584 * 24,437 t = 13,646 t
630	0.1943 * 24,437 t = 4,748 t

4) For the B season, the allocation of TAC to areas 610, 620 and 630 is based on the "composite" estimate of winter biomass distribution

610	0.2473 * 24,437 t = 6,043 t
620	0.6646 * 24,437 t = 16,241 t
630	0.0881 * 24,437 t = 2,153 t

5) For the C and D seasons, the allocation of remaining TAC to areas 610, 620 and 630 is based on the average biomass distribution in areas 610, 620 and 630 in the most recent four NMFS bottom trawl surveys.

610	0.4595 / (1 – 0.0225) x 24,437 = 11,487 t
620	0.2244 / (1 – 0.0225) x 24,437 = 5,610 t
630	0.2937 / (1 – 0.0225) x 24,437 = 7,342 t

610	0.4595 / (1 – 0.0225) x 24,437 = 11,487 t
620	0.2244 / (1 – 0.0225) x 24,437 = 5,610 t
630	0.2937 / (1 – 0.0225) x 24,437 = 7,342 t

Appendix Table 1. Estimates of percent pollock in management areas 610-630 during winter EIT surveys in the Gulf of Alaska.

Survey	Year				Percent of biomass by management area			Percent of total biomass		
		Model estimates of total 2+ biomass at spawning	Survey biomass estimate[1]	Percent	Area 610	Area 620	Area 630	Area 610	Area 620	Area 630
Shelikof Strait	1992	1,018,880	681,400	66.9%						
Shelikof Strait	1993	1,128,430	408,200	36.2%						
Shelikof Strait	1994	1,127,330	467,300	41.5%						
Shelikof Strait	1995	935,423	618,300	66.1%						
Shelikof Strait	1996	831,504	745,400	89.6%						
Shelikof Strait	1997	750,855	570,100	75.9%	0.0%	98.8%	1.2%			
Shelikof Strait	1998	617,717	489,900	79.3%	0.0%	97.5%	2.5%			
Shelikof Strait	2000	488,167	334,900	68.6%	0.0%	97.8%	2.2%			
Shelikof Strait	2001	543,573	369,600	68.0%	0.0%	98.3%	1.7%			
Shelikof Strait	2002	588,229	229,104	38.9%	0.0%	97.7%	2.3%			
Shelikof Strait	Average			63.1%	0.0%	98.0%	2.0%	0.0%	61.9%	1.2%
Shumagin	1995	935,423	290,100	31.0%	90.0%	10.0%	0.0%			
Shumagin	2001	543,573	108,791	20.0%	84.8%	15.2%	0.0%			
Shumagin	2002	588,229	135,644	23.1%	100.0%	0.0%	0.0%			
Shumagin	Average			24.7%	91.6%	8.4%	0.0%	22.6%	2.1%	0.0%
Shelf break/east side Kodiak	1990	1,042,640	96,610	9.3%	14.9%	6.2%	78.9%	1.4%	0.6%	7.3%
Total				97.06%				24.00%	64.51%	8.55%
Rescaled total				100.00%				24.73%	66.46%	8.81%

[1]The biomass of age-1 pollock was not included in Shelikof Strait survey biomass in 1995 and 2000.

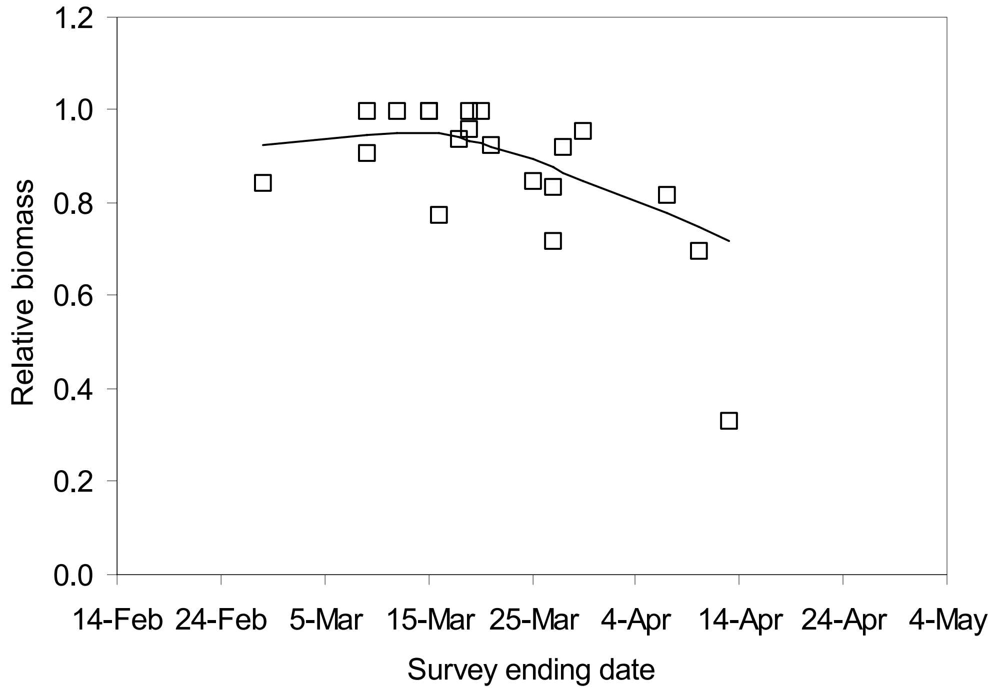
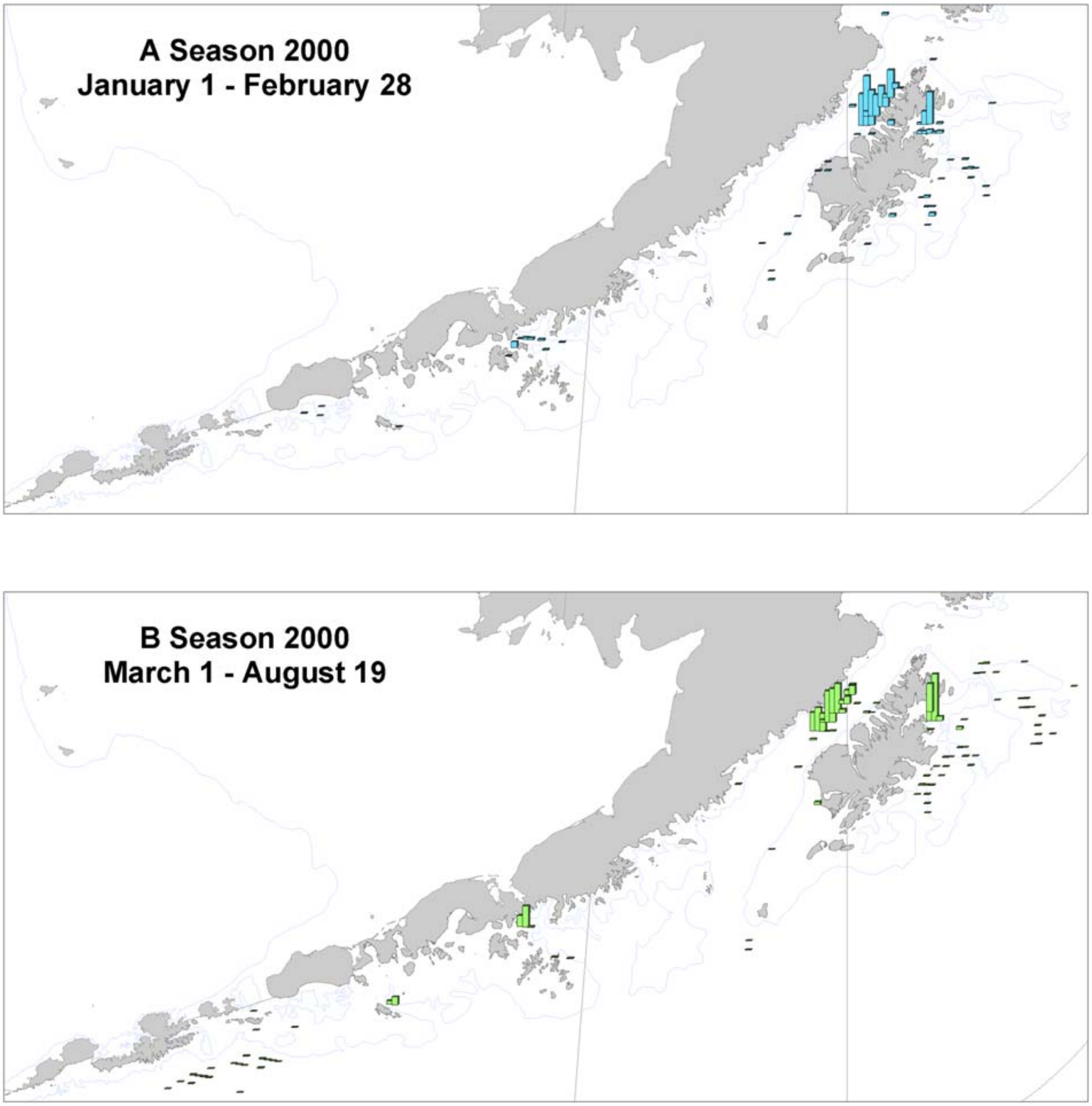
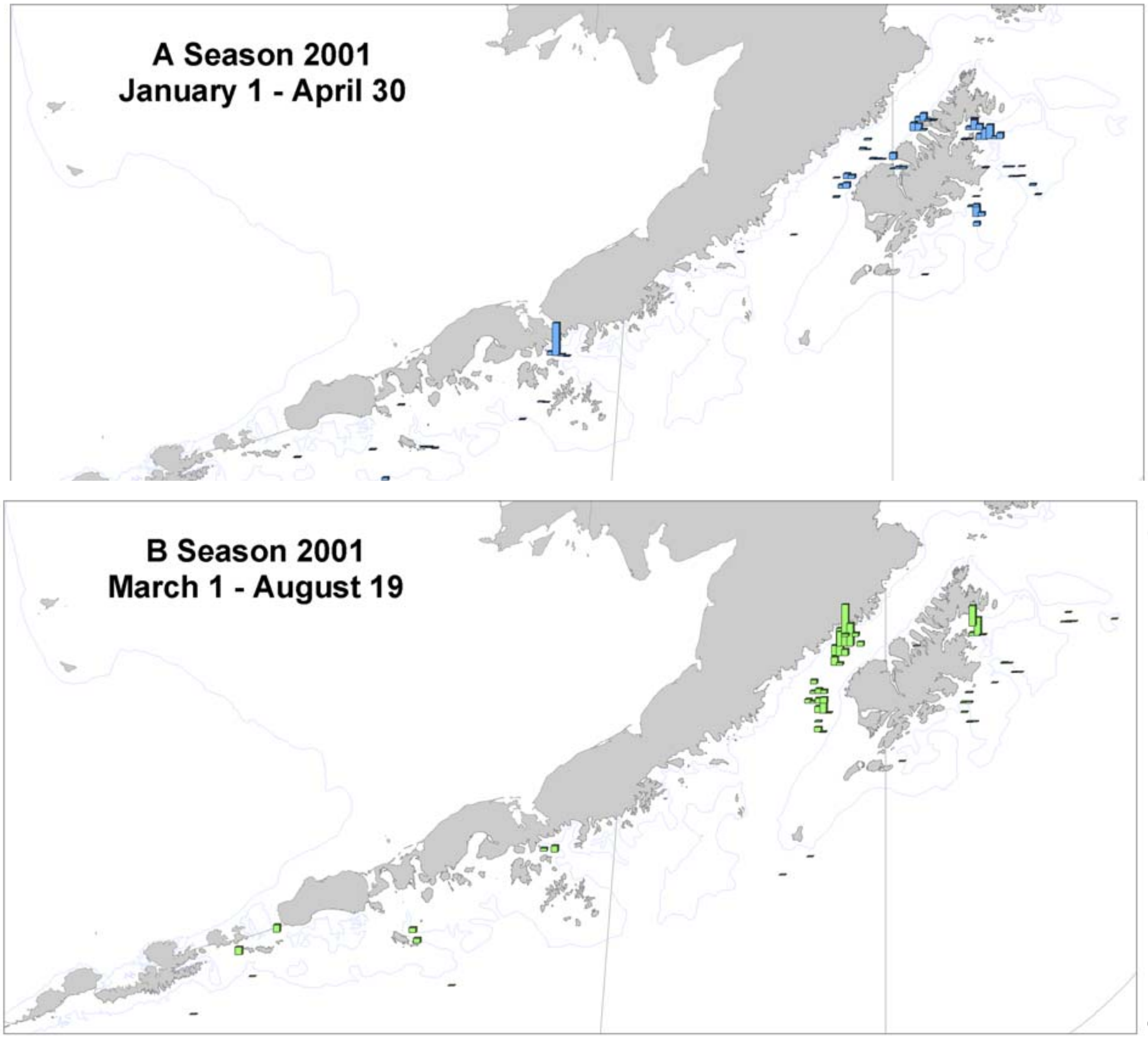


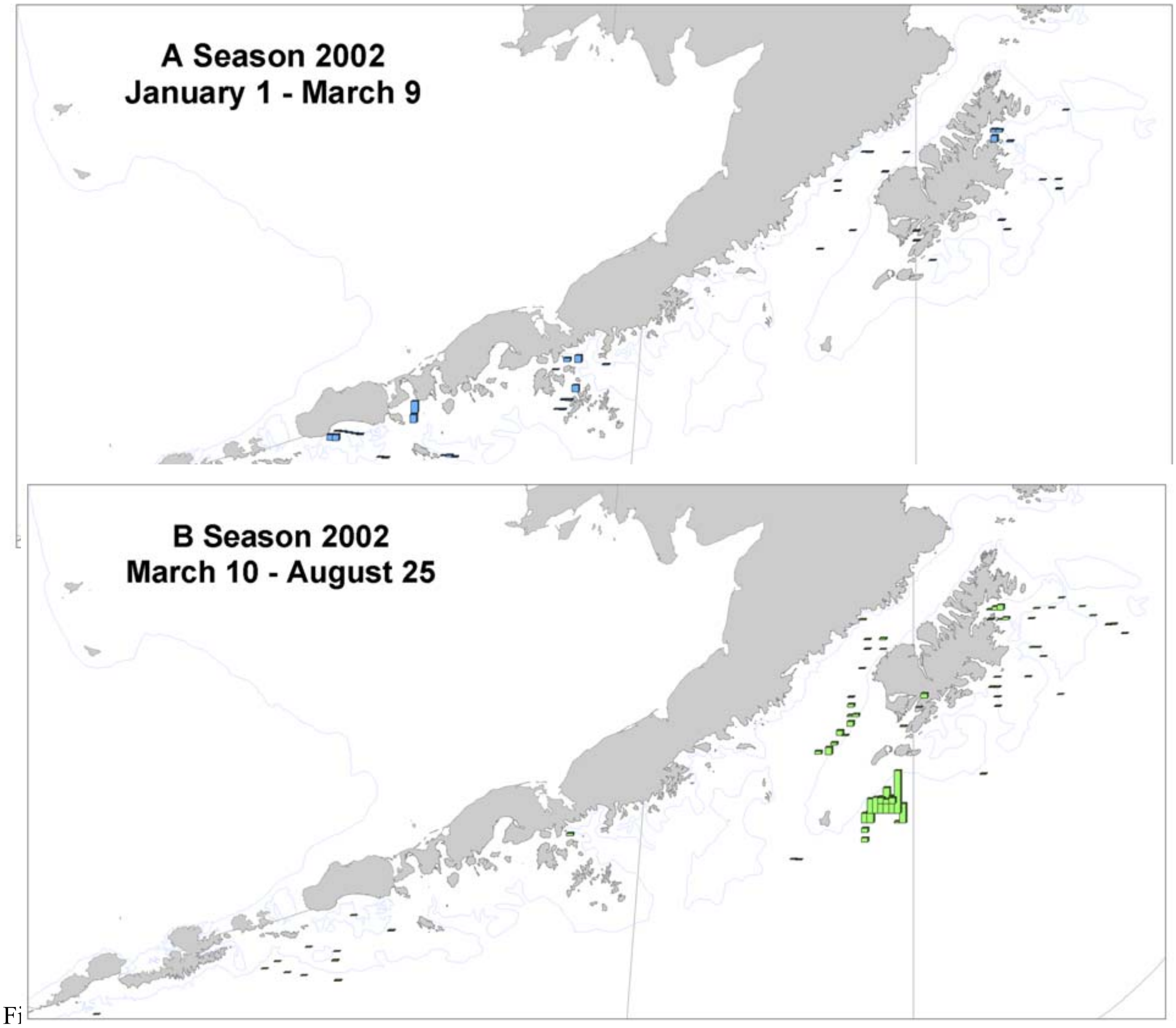
Figure 28. Relative biomass of pollock in Shelikof Strait as a function of survey ending date from replicated acoustic surveys in 1981-88. Relative biomass is the survey biomass divided by the maximum survey biomass for the year. A lowess smooth is also shown.


A Season 2000
January 1 - February 28
B Season 2000
March 1 - August 19

Fi


A Season 2001
January 1 - April 30
B Season 2001
March 1 - August 19

01.


A Season 2002
January 1 - March 9
B Season 2002
March 10 - August 25

Fi 2.

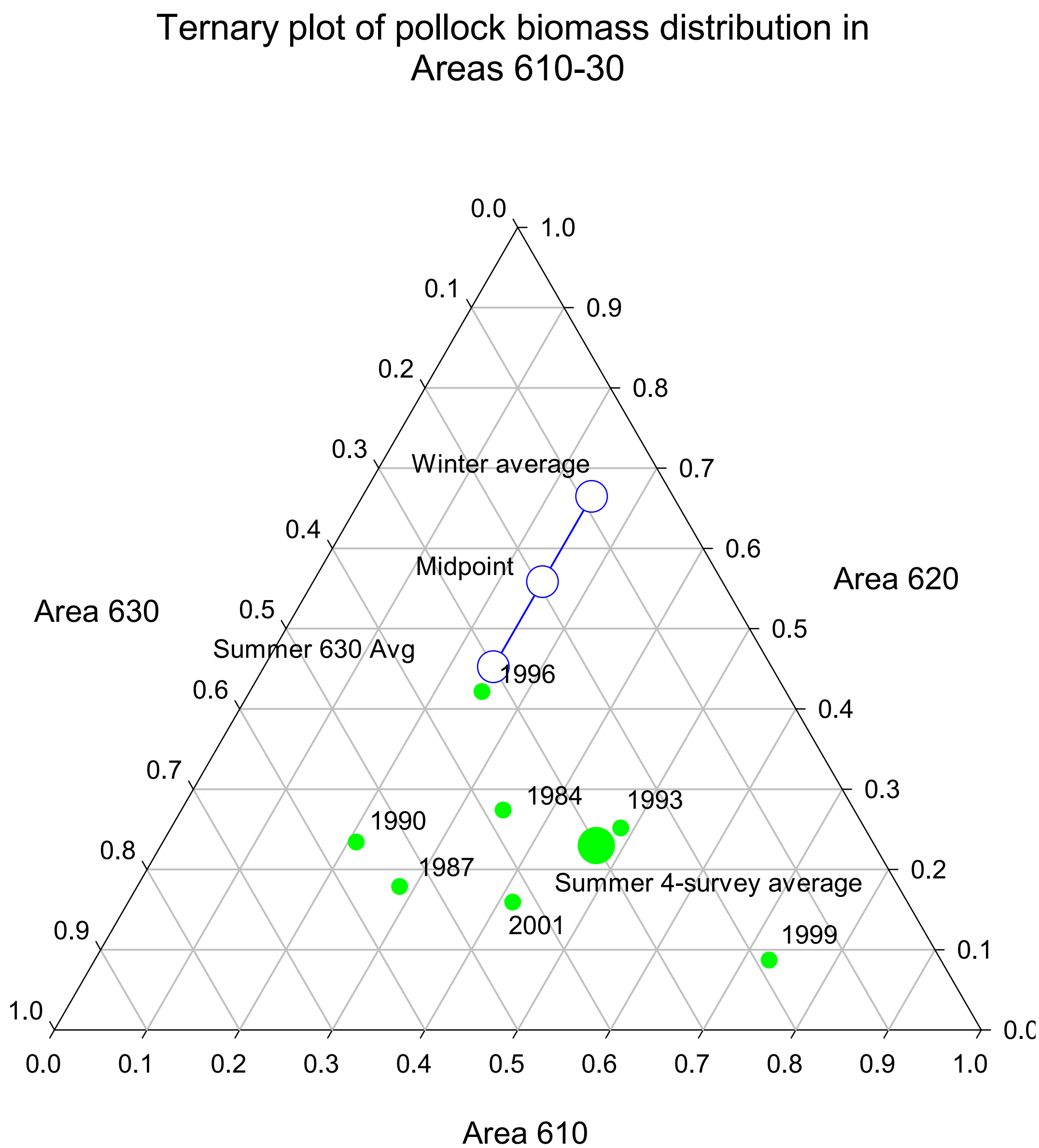
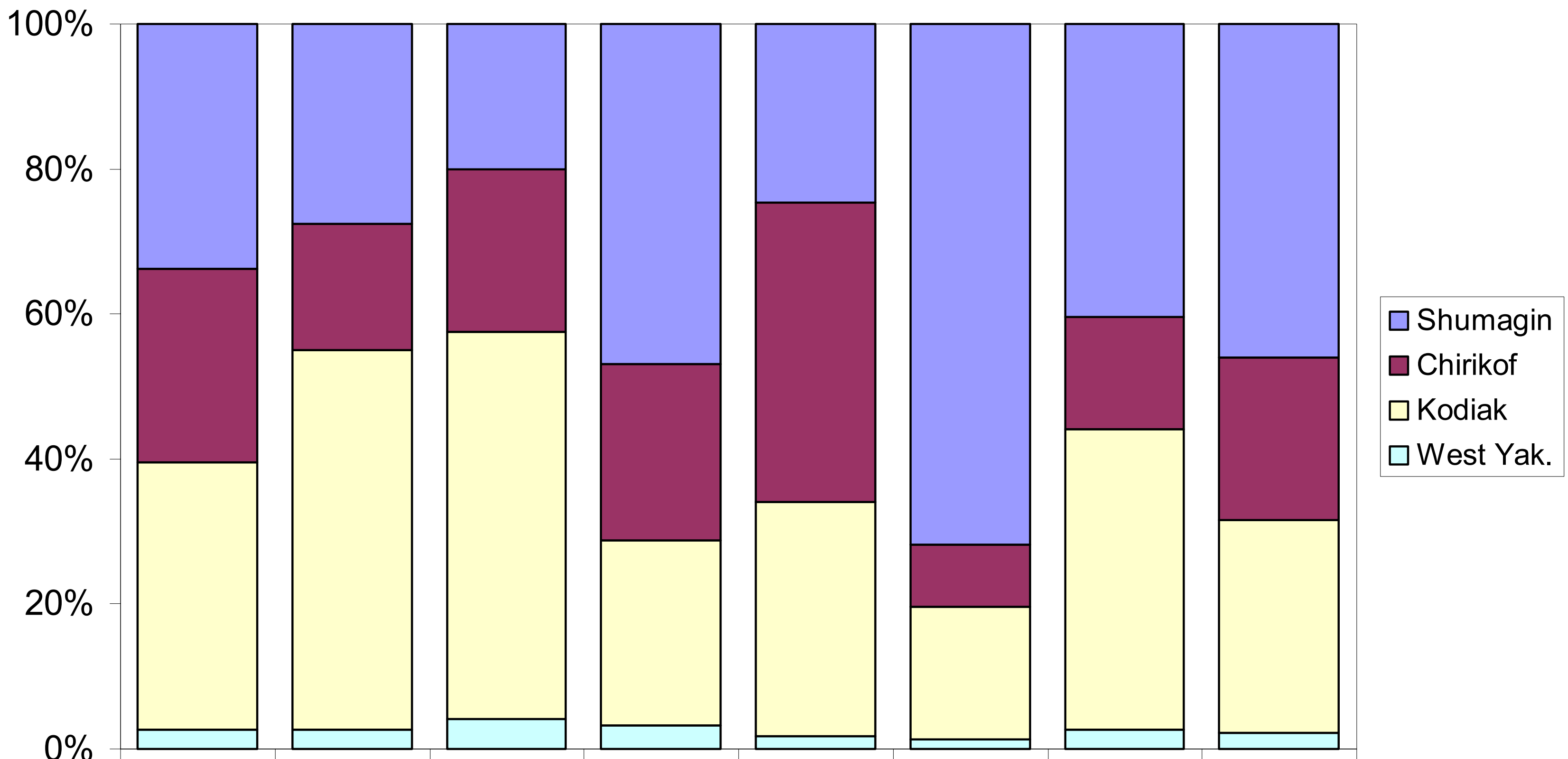


Figure 32. Ternary plot of the seasonal biomass distribution of walleye pollock in the Gulf of Alaska.



	1984	1987	1990	1993	1996	1999	2001	4 survey avg (1993-2001)
Shumagin	33.82%	27.59%	20.03%	46.98%	24.60%	71.81%	40.40%	45.95%
Chirikof	26.66%	17.38%	22.41%	24.32%	41.37%	8.57%	15.49%	22.44%
Kodiak	36.90%	52.41%	53.43%	25.49%	32.20%	18.30%	41.49%	29.37%
West Yak.	2.62%	2.62%	4.13%	3.21%	1.83%	1.32%	2.62%	2.25%

Figure 33. Percent distribution of Gulf of Alaska pollock biomass west of 140° W lon. in NMFS bottom trawl surveys in 1984-2001. The percent in West Yakutat in 1984, 1987, and 2001 was set equal to the mean percent in 1990-99.

Appendix C: Gulf pollock stock assessment model

Population dynamics: The age-structured model for pollock describes the relationships between population numbers by age and year. The modeled population includes individuals from age 2 to age 10, with age 10 defined as a "plus" group, i.e., all individuals age 10 and older. The model extends from 1964 to 1999 (36 yrs). The Baranov (1918) catch equations are assumed, so that

$$c_{ij} = N_{ij} \frac{F_{ij}}{Z_{ij}} [1 - \exp(-Z_{ij})]$$

$$N_{i+1\,j+1} = N_{ij} \exp(-Z_{ij})$$

$$Z_{ij} = \sum_k F_{ij} + M$$

except for the plus group, where

$$N_{i+1,10} = N_{i,9} \exp(-Z_{i,9}) + N_{i,10} \exp(-Z_{i,10})$$

where N_{ij} is the population abundance at the start of year *i* for age *j* fish, F_{ij} = fishing mortality rate in year *i* for age *j* fish, and c_{ij} = catch in year *i* for age *j* fish. A constant natural mortality rate, *M*, irrespective of year and age, is assumed.

Fishing mortality is modeled as a product of year-specific and age-specific factors (Doubleday 1976)

$$F_{ij} = s_j f_i$$

where s_j = age-specific selectivity, and f_i = the annual fishing mortality rate. To ensure that the selectivities are well determined, we require that $\max(s_j) = 1$. Following previous assessments, a scaled double-logistic function (Dorn and Methot 1990) was used to model age-specific selectivity,

$$s'_j = \left(\frac{1}{1 + \exp[-\beta_1(j - \alpha_1)]} \right) \left(1 - \frac{1}{1 + \exp[-\beta_2(j - \alpha_2)]} \right)$$

$$s_j = s'_j / \max(s'_j)$$

where α_1 = inflection age, β_1 = slope at the inflection age for the ascending logistic part of the equation, and α_2 , β_2 = the inflection age and slope for the descending logistic part.

Measurement error

Model parameters were estimated by maximum likelihood (Fournier and Archibald 1982, Kimura 1989, 1990, 1991). Fishery observations consist of the total annual catch in tons, C_i, and the proportions at age in the catch, p_{ij} . Predicted values from the model are obtained from

$$\hat{C}_i = \sum_j w_{ij}\ c_{ij}$$

$$\hat{p}_{ij} = c_{ij} / \sum_j c_{ij}$$

where w_{ij} is the weight at age j in year i . Year-specific weights at age are used when available.

Log-normal measurement error in total catch and multinomial sampling error in the proportions at age give a log-likelihood of

$$\log L_k = \sum_i [\ \log(\ C_i\ ) - \log(\ \hat{C}_i\ )\ ]^2 / 2\ \sigma_i^2 + \sum_i m_i \sum_j p_{ij} \log(\ \hat{p}_{ij} / p_{ij}\ )$$

where σ_i is standard deviation of the logarithm of total catch (~ CV of total catch) and m_i is the size of the age sample. In the multinomial part of the likelihood, the expected proportions at age have been divided by the observed proportion at age, so that a perfect fit to the data for a year gives a log likelihood value of zero (Fournier and Archibald 1982). This formulation of the likelihood allows considerable flexibility to give different weights (i.e. emphasis) to each estimate of annual catch and age composition. Expressing these weights explicitly as CVs (for the total catch estimates), and sample sizes (for the proportions at age) assists in making reasonable assumptions about appropriate weights for estimates whose variances are not routinely calculated.

Survey observations consist of a total biomass estimate, B_i, and survey proportions at age π_{ij}. Predicted values from the model are obtained from

$$\hat{B}_i = q \sum_j w_{ij}\ s_j\ N_{ij} \exp [\ \phi_i\ Z_{ij}\ ]$$

where q = survey catchability, w_{ij} is the survey weight at age *j* in year *i* (if available), s_j = selectivity at age for the survey, and ϕ_i = fraction of the year to the mid-point of the survey. Although there are multiple surveys for Gulf pollock, a subscript to index a particular survey has been suppressed in the above and subsequent equations in the interest of clarity. Survey selectivity was modeled using a either a double-logistic function of the same form used for fishery selectivity, or simpler variant, such as single logistic function. The expected proportions at age in the survey in the *i*th year are given by

$$\hat{\pi}_{ij} = s_j\, N_{ij} \exp[\phi_i Z_{ij}] \Big/ \sum_j s_j\, N_{ij} \exp[\phi_i Z_{ij}]$$

Log-normal errors in total biomass and multinomial sampling error in the proportions at age give a log-likelihood for survey *k* of

$$\log L_k = \sum_i [\log(B_i) - \log(\hat{B}_i)]^2 / 2\sigma_i^2 + \sum_i m_i \sum_j \pi_{ij} \log(\hat{\pi}_{ij} / \pi_{ij})$$

where σ_i is the standard deviation of the logarithm of total biomass (~ CV of the total biomass) and m_i is the size of the age sample from the survey.

Process error

Process error refers to random changes in parameter values from one year to the next. Annual variation in recruitment and fishing mortality can be considered types of process error (Schnute and Richards 1995). In the pollock model, these annual recruitment and fishing mortality parameters are generally estimated as free parameters, with no additional error constraints. We use a process error to describe changes in fisheries selectivity over time. To model temporal variation in a parameter γ , the year-specific value of the parameter is given by

$$\gamma_i = \bar{\gamma} + \delta_i$$

where $\bar{\gamma}$ is the mean value (on either a log scale or linear scale), and δ_i is an annual deviation subject to the constraint $\sum \delta_i = 0$. For a random walk where annual *changes* are normally distributed, the log-likelihood is

$$\log L_{Proc.\,Err.} = \sum \frac{(\delta_i - \delta_{i+1})^2}{2\sigma_i^2}$$

where σ_i is the standard deviation of the annual change in the parameter. We use a process error model for all four parameters of the fishery double-logistic curve.

The total log likelihood is the sum of the likelihood components for each fishery and survey, plus a term for process error,

$$\text{Log}\,L = \sum_{k} \text{Log}\,L_k + \sum_{p} \text{Log}\,L_{Proc.\,Err.}\,.$$